Debt Financing and Real Output Growth: Is There a Threshold Effect?

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Introduction

- The relationship between public debt expansion and economic growth has attracted a lot of interest in recent years.
- This presentation focuses on the following issues:
  - Whether a debt-GDP threshold exists and is of consequence for macro policy.
  - Evidence on conditions (shock scenarios) under which increases in debt-to-GDP do or do not result in growth slow downs.

This talk is based on the following papers:

- Analysis of **threshold effects** and **long-run** relationships:
  - A. Chudik, K. Mohaddes, M. H. Pesaran, and M. Raissi (2017, CMPR) Is There a Debt-threshold Effect on Output Growth?, *Review of Economics and Statistics*, 99, 135-150.
  - A. Chudik, K. Mohaddes, M. H. Pesaran, and M. Raissi (2016, CMPR), Long-Run Effects in Large Heterogeneous Panel Data Models with Cross-Sectionally Correlated Errors, *Advances in Econometrics*, 36, Essays in Honor of Aman Ullah, 85-135.

- A. Chudik, K. Mohaddes, M. H. Pesaran (2017, CMP), Global and Country-Specific Effects of Technology and Fiscal Policy Shocks on Output and Debt using A GVAR Model, work in progress.

Literature

- The predictions of the theoretical literature on the long-run effects of public debt on output growth are ambiguous, predicting a negative as well as a positive effect under certain conditions. More on this later.
- Sustainability of sovereign debt requires a stable (stationary) debt-to-GDP ratio in the very long run, but there are clear evidence of prolonged periods of imbalance between debt and GDP, particularly in the case of industrialized economies.
- Large increases in debt-to-GDP ratio experienced by US and many European economies in the aftermath of 2008 financial crises has led some researchers, in particular Reinhart and Rogoff (2010), to argue for a non-linear relationship, characterized by a threshold effect, between public debt and output growth.

- RR do not provide a formal statistical framework, and simply bin annual observations on country-specific growth rates by debt-to-GDP ratio into four groups -those with debt-to-GDP falling below 30%, between 30% and 60%, between 60% and 90%, and above 90%, and concludes that countries with debt-to-GDP above 90% tend to have a lower average and median growth rates.

- The analysis of RR has generated a considerable degree of debate in the literature. See, for example, Woo and Kumar (2015), Checherita-Westphal and Rother (2012), Eberhardt and Presbitero (2015), and Reinhart et al. (2012). Panizza and Presbitero (2013) provide a survey.

- These studies address a number of important modelling issues not considered by RR, but they nevertheless either employ panel data models that impose slope homogeneity and/or do not adequately allow for cross-sectional dependence across individual country errors.

- It is further implicitly assumed that different countries converge to their equilibrium at the same rate, and there are no spillover effects of debt overhang from one country to another.

- In our research (CMPR) we investigate the issue of the debt threshold using a cross-country dynamic panel that allow for endogeneity of debt and growth, fixed effects, slope heterogeneity, and cross-sectional error dependence.

A panel threshold output growth model

- We consider two threshold variables - a standard threshold variable:

$$g_1(d_{it}, \tau) = I\left[d_{it} > \ln(\tau)\right],$$

and our research uncovers that we also need to consider the interactive threshold variable:

$$g_2(d_{it}, \tau) = I\left[d_{it} > \ln(\tau)\right] \times \max(0, \Delta d_{it}),$$

which takes a non-zero value only if d_{it} exceeds the threshold **and** debt-to-GDP is rising.

- We treat the threshold, τ, as an unknown parameter, and develop a test of the threshold effect ($H_0 : \boldsymbol{\varphi} = (\varphi_1, \varphi_2)' = \mathbf{0}$) using the following threshold ARDL panel data model

$$\begin{aligned} \Delta y_{it} &= c_i + \varphi_1 g_1(d_{it}, \tau) + \varphi_2 g_2(d_{it}, \tau) + \lambda_i \Delta y_{i,t-1} \\ &\quad + \beta_{i0} \Delta d_{it} + \beta_{i1} \Delta d_{i,t-1} + \beta_{i2} d_{i,t-1} + u_{it}, \end{aligned} \tag{1}$$

for $i = 1, 2, ..., N$, and allow for common factors and cross sectional error dependence.

- It is important to allow for heterogeneity of slope coefficients, since even if the underlying threshold VAR specification for output and debt had homogenous slopes, the threshold ARDL panel data model will feature heterogenous slopes due to possible correlations between the innovations of the output and debt equations.

- The parameters are estimated using cross-section augmented ARDL and DL methods (CS-ARDL, and CS-DL, respectively), which deal with unobserved common factors (Chudik and Pesaran, 2015, and CMPR)

Data

Our database features the CPI, real GDP and gross government debt/GDP data series for an unbalanced panel of 40 countries covering the sample period 1965-2010, with $T_{\min} = 30$, and $N_{\min} = 20$ across all countries and time periods.

Europe	MENA Countries	Asia Pacific	Latin America
Austria	Egypt	Australia	Argentina
Belgium	Iran	China	Brazil
Finland	Morocco	India	Chile
France	Syria	Indonesia	Ecuador
Germany	Tunisia	Japan	Peru
Italy	Turkey	Korea	Venezuela
Netherlands		Malaysia	
Norway	**North America**	New Zealand	**Rest of Africa**
Spain	Canada	Philippines	Nigeria
Sweden	Mexico	Singapore	South Africa
Switzerland	United States	Thailand	
United Kingdom			

- The CPI and real GDP data series are from the IMF *International Financial Statistics* database except for CPI data for Brazil, China and Tunisia which is from the IMF *World Economic Outlook* database and CPI data for UK which is from the Reinhart and Rogoff's *Growth in a Time of Debt* database.
- The gross government debt/GDP data series are from Reinhart and Rogoff (2011) and their most-up-to date *From Financial Crash to Debt Crisis* online database, except for Iran, Morocco, Nigeria, and Syria for which the IMF FAD *Historical Public Debt* database was used instead.
- We focus on gross debt data due to difficulty of collecting net debt data on a consistent basis over time and across countries. Moreover, we use public debt at the general government level for as many countries as possible.

Table 1: Evidence of standard threshold effects

Threshold definition:	$g_1(d_{it}, \tau) = I[d_{it} > \ln(\tau)]$				
Estimation method:	CS-ARDL		CS-DL		
Maximum lag order:	1	2	0	1	2
Estimated threshold level:	40%	30%	40%	40%	40%
	Statistical significance of the threshold effect (at 5% or 1%)				
Based on $Sup\mathcal{T}$ test	no	no	no	no	no
Based on $Ave\mathcal{T}$ test	no	no	no	no	no

- No evidence is found for a universally applicable threshold effect in the relationship between public debt and economic growth.

Table 2: Evidence of an interactive threshold effects

Threshold definition:	$g_2(d_{it}, \tau) = I\left[d_{it} > \ln(\tau)\right] \times \max(0, \Delta d_{it})$				
Estimation method:	CS-ARDL		CS-DL		
Maximum lag order:	1	2	0	1	2
Estimated threshold level:	60%	60%	60%	60%	60%
	Statistical significance of the threshold effect (at 5% or 1%)				
Based on $Sup\mathcal{T}$ test	no	no	no	yes: 5%	yes: 5%
Based on $Ave\mathcal{T}$ test	yes: 1%	yes: 1%	yes: 1%	yes: 1%	yes: 1%

- Countries with rising debt-to-GDP ratios beyond 60% tend to have lower real output growth rates, although the evidence weakens when we consider advanced economies separately from the emerging economies.

Evidence on conditions (shock scenarios) under which increases in debt-to-GDP do or do not result in growth slow downs.

Dynamics of Public Debt and Long Run Equilibrium Relationship between Debt and GDP

- The process of debt accumulation is governed by

$$D_t = (1 + r_t) D_{t-1} + PD_t,$$

where D_t is the real debt outstanding, r_t is the real interest on debt, and PD_t is the real primary deficit in period t. Dividing both sides by Y_t we have

$$\left(\frac{D}{Y}\right)_t = \left(\frac{1 + r_t}{1 + g_t}\right) \left(\frac{D}{Y}\right)_{t-1} + \left(\frac{PD}{Y}\right)_t .$$

Debt sustainability requires that $\phi_t = PD_t / Y_t$ is stable (stationary) and the long run average growth rate $(T^{-1} \Sigma_{\tau=1}^{T} g_\tau)$ is strictly larger than the average rate of interest on debt $\left(T^{-1} \Sigma_{\tau=1}^{T} r_\tau\right)$. Under these conditions $\log (D/Y)_t = d_t - y_t$ must be stationary.

Evidence of cointegration properties of d_t and y_t

- As result all general equilibrium models with balanced growth paths and government debt financing require that log of real output (y_t) and log of real debt (d_t) are cointegrated with unit coefficient, namely

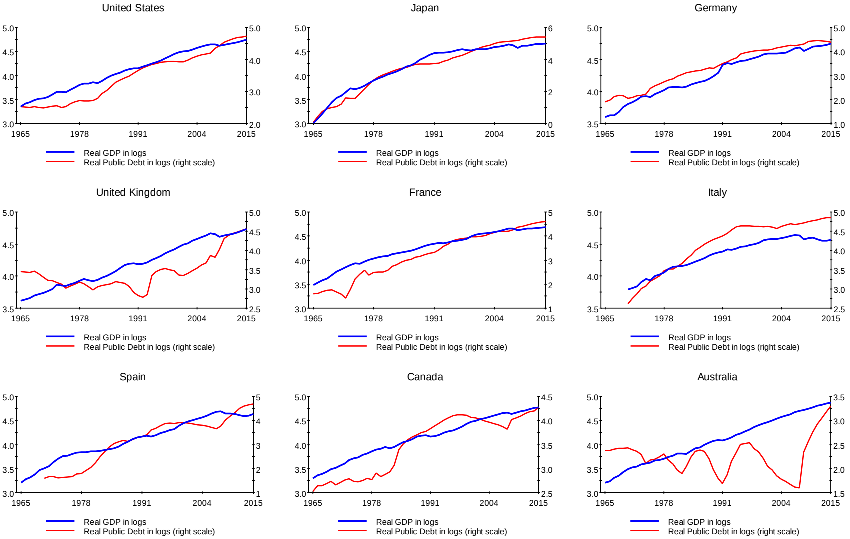
$$y_t = \mu + d_t + \xi_t,$$

  where ξ_t is a mean zero stationary process.

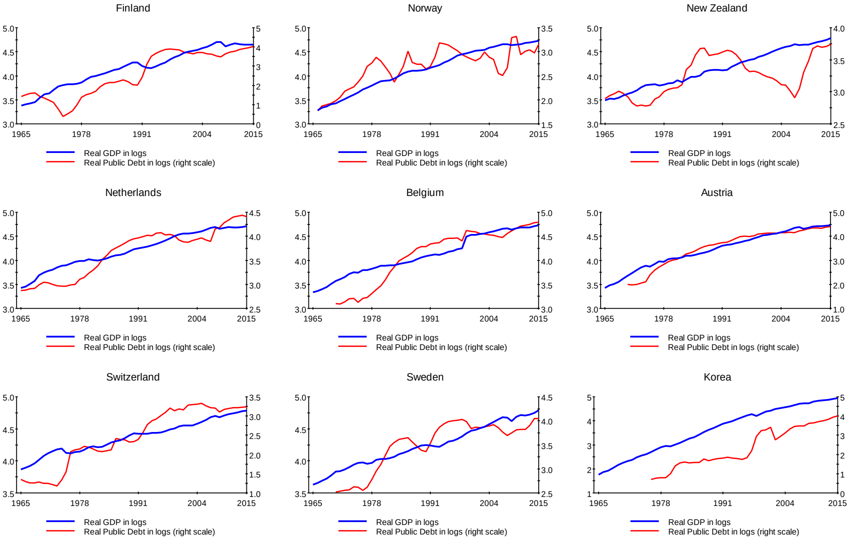
- However, time-horizon for this theoretical long-run relationship can be very long (more than the available sample of few decades of data).

- In what follows we provide graphic and statistical tests of the relationship between y_{it} and d_{it} across a number of advanced and emerging economies.

Plots of Real GDP and Public Debt (right scale), in logs



Plots of Real GDP and Public Debt (right scale), in logs



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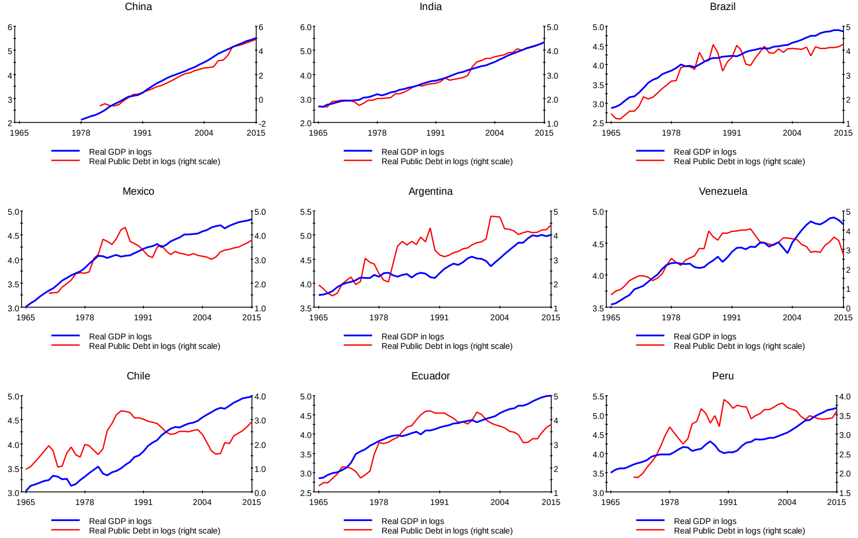
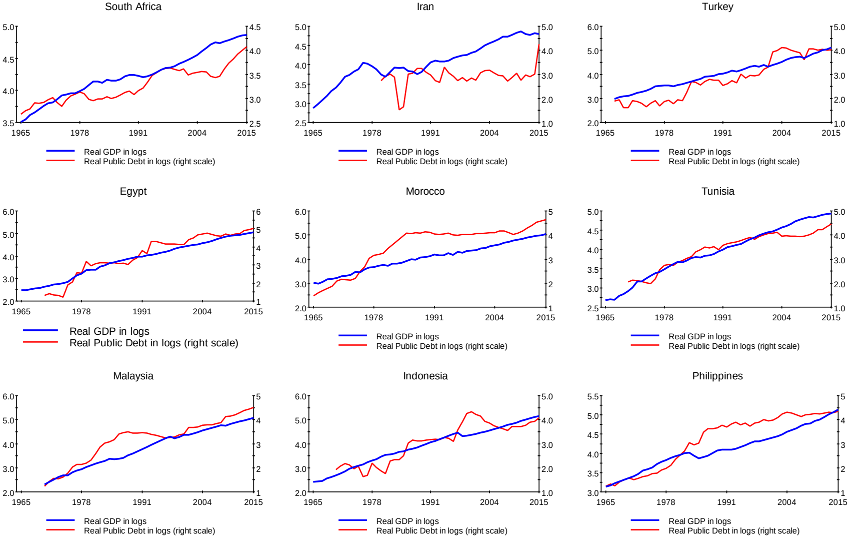


Plots of Real GDP and Public Debt (right scale), in logs



- In the horizon of 'only' few decades of data, a cointegration analysis of y_{it} and d_{it} cannot be detected statistically for about half of the countries in the sample.

- When cointegration is detected, then it is not necessarily the case that $y_{it} = \mu_i + \beta_i d_{it} + \xi_{it}$ with $\beta_i = 1$.

- In the steady state, we must have $\beta_i = 1$, otherwise a balanced growth path cannot exist. But in medium-to-long run d_{it} and y_{it} need not cointegrate and/or β_i might differ from unity.

- Taking first-differences (to avoid the issue of non-cointegration) we obtain

$$\Delta \left(d_{it} - y_{it} \right) = \gamma_i \Delta y_{it} + \Delta \xi_{it}, \text{ where } \gamma_i = \left(1 - \beta_i \right) / \beta_i. \quad (2)$$

- When $\beta_i \neq 1$ ($\gamma_i \neq 0$), two possibilities can arise:
  - $\beta_i < 1$ ($\gamma_i > 0$), in which case an increase in output is associated with a deterioration in the debt-to-GDP ratio
  - $\beta_i > 1$ ($\gamma_i < 0$), in which case an increase in output is associated with an improvement in the debt-to-GDP ratio.

- BUT it is important to note that neither of the above possibilities are sustainable - countries with $\beta_i < 1$ must eventually switch to having $\beta_i > 1$, and *vice versa*.

- In addition to distinguishing between short-term and long-term effects, it is important that global factors are also taken into account.
- This requires a multi-country approach that allows for a sufficient degree of heterogeneity across countries.
- We shall be using the GVAR approach to model dynamic relationship between the debt and growth, focusing on short-run (business cycle) effects.
- GVAR approach was introduced by Pesaran et al. (2004), and has been developed further and applied in numerous empirical applications. See Pesaran (2015) for a textbook treatment.

GVAR approach to modeling the global economy

- Let $\mathbf{x}_{it}$ be $k \times 1$ vector of variables in country i, and assume that $\mathbf{x}_{it}$ is given by the following country factor-augmented VAR (FAVAR) model,

$$\mathbf{x}_{it} - \boldsymbol{\Gamma}_i \mathbf{f}_t = \boldsymbol{\Phi}_i \left(\mathbf{x}_{it-1} - \boldsymbol{\Gamma}_i \mathbf{f}_{t-1} \right) + \mathbf{e}_{it}, \; i = 1, 2, ..., N \qquad (3)$$

  where $\mathbf{e}_{it}$ is vector of country-specific residuals allowed to be weakly correlated across countries and $\mathbf{f}_t$ is a $m \times 1$ vector of unobserved common factors given by a VAR model,

$$\mathbf{f}_t = \boldsymbol{\Psi} \mathbf{f}_{t-1} + \boldsymbol{\eta}_t. \qquad (4)$$

- Specification (3)–(4) is convenient for illustrative purposes, and it can be generalised in a number of important ways, including (i) the way factors enter the country models, (ii) inclusion of deterministic trends, and (iii) higher order lags.

- The unobserved factors can be approximated by PCs of cross-section averages,

$$\bar{\mathbf{x}}_t = N^{-1} \sum_{i=1}^{N} \mathbf{x}_{it},$$

and the reduced form common shocks $\mathbf{v}_t = E\left(\boldsymbol{\Gamma}_i\right) \boldsymbol{\eta}_t$, can be recovered from VAR model in $\bar{\mathbf{x}}_t$,

$$\bar{\mathbf{x}}_t = \boldsymbol{\Psi}\, \bar{\mathbf{x}}_{t-1} + \mathbf{v}_t + O_p\left(N^{-1/2}\right).$$

- The reduced form common shocks $\mathbf{v}_t$ are identified for N sufficiently large, but the common factors, $\mathbf{f}_t$, and the associated global shocks, $\boldsymbol{\eta}_t$, are not.
- For identification of common shocks, a suitable rotation $\boldsymbol{\varepsilon}_t = \mathbf{A}\mathbf{v}_t$ (based on economic theory considerations) could be considered.
- For a given $\mathbf{A}$, we can recover $\hat{\boldsymbol{\varepsilon}}_t$, and estimate augmented country-specific VARs,

$$\mathbf{x}_{it} = \boldsymbol{\Phi}_{ii}\mathbf{x}_{i,t-1} + \mathbf{H}_i\hat{\boldsymbol{\varepsilon}}_t + \mathbf{B}_i\bar{\mathbf{x}}_{t-1} + \mathbf{e}_{it} + O_p\left(N^{-1/2}\right). \quad (5)$$

  Fit of the above specification does not depend on the identification of common shocks (the number of lags of $\bar{\mathbf{x}}_t$ in country models cannot be smaller than the number of lags used in the marginal VAR model for $\bar{\mathbf{x}}_t$).
- For identification of country-specific idiosyncratic shocks with an economic interpretation, a suitable rotation $\boldsymbol{\epsilon}_{it} = \mathbf{A}_i\mathbf{e}_{it}$ could be considered.

Short-run effects: Impulse Response Functions

- Identifying assumptions for the distinction between global (common) and local (idiosyncratic) shocks are build-in the specification of our model, where each country-specific model conditions on the global shocks/variables.
- In order to identify shocks (global and country-specific ones), further assumptions are required.
- One possibility is to follow the macro literature and use sign restrictions.

Identification of technology (output) and fiscal shocks

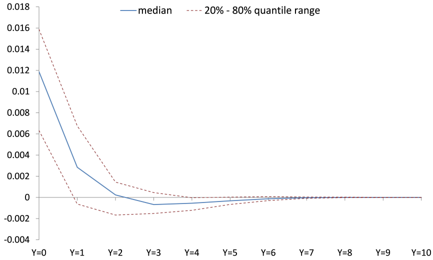
- Over the business cycle a technology shock is expected to increase output (on impact) without adversely affecting debt; whilst a fiscal shock (expansion) is likely to increase output (on impact) with adverse effects on debt.

- This is similar to the identification of demand and supply shocks by sign restrictions. The Bayesian sign identification of Baumeister and Hamilton (2015) can be used for this purpose (to be implemented).

- Alternatively, Cholesky ordering can be used to identify the two types of shocks by assuming that the technology shock affects both growth and debt-to-GDP on impact, but fiscal shock affects output with a lag.

Chart 1: Impulse response function for the effects of positive global technology (output) shock across countries

real output growth



growth in debt-to-GDP ratio

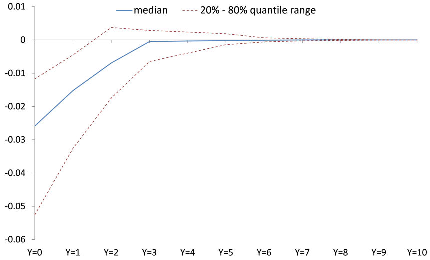
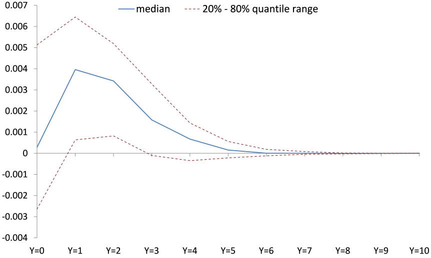


Chart 2: Impulse response function for the effects of positive global fiscal shock across countries

real output growth



growth in debt-to-GDP ratio

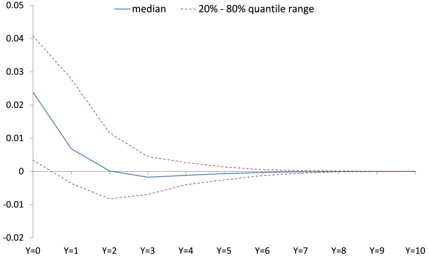


Chart 3: Impulse response function for the effects of positive domestic technology (output) shock across countries

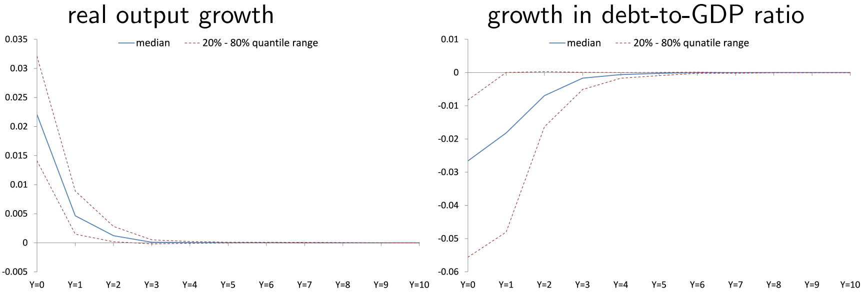
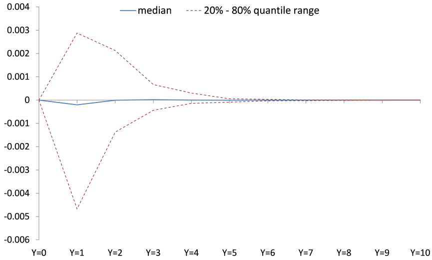
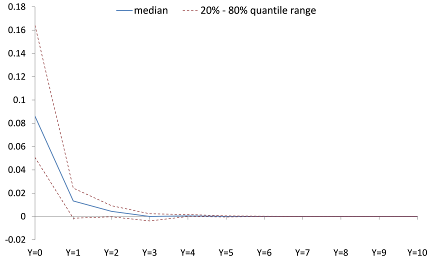


Chart 4: Impulse response function for the effects of positive domestic fiscal shock across countries

real output growth



growth in debt-to-GDP ratio



Main Takeaways from Impulse-Response Findings

- Effects of all shocks tend to dissipate within 4–5 years. Effects of technology (output) shocks are more persistent.
- Global fiscal shock appear more effective in stimulating output than country fiscal shocks. This points to benefits of coordination of fiscal policies across countries.
- Global and country-specific technology (output) shocks have similar effect - both contribute to a decline in debt-to-GDP ratio.

Importance of individual shocks: Forecast Error Variance Decompositions (FEVD)

- In order to illustrate importance of individual shocks, we compute the standard Forecast Error Variance Decompositions.
- Findings for the overall effects of global shocks vis-a-vis the group of idiosyncratic shocks does not depend on the Cholesky identification ordering.
- The distinction between the technology (output) and fiscal shocks, on the other hand, depends on the identification employed.
- Technology (output) shock explain 18% to 29% of variation in debt-to-GDP ratio and about 82%-97% of output fluctuations, depending on the horizon (medians across countries).

FEVD: Global and local shocks (medians across countries)

	output growth			
	Y=0	Y=1	Y=5	Y=10
Global shocks	24.3%	27.4%	30.9%	31.1%
Domestic idiosyncratic shocks	71.9%	63.2%	58.2%	58.1%
	debt-to-GDP growth			
	Y=0	Y=1	Y=5	Y=10
Global shocks	12.2%	17.2%	20.7%	20.8%
Domestic idiosyncratic shocks	77.5%	67.1%	62.8%	62.8%

FEVD: Technology (output) and fiscal shocks (medians across countries)

	output growth			
	Y=0	Y=1	Y=5	Y=10
Technology (output) shocks	96.8%	87.8%	81.9%	81.8%
Fiscal shocks	1.4%	8.7%	12.6%	12.7%
	debt-to-GDP growth			
	Y=0	Y=1	Y=5	Y=10
Technology (output) shocks	18.2%	28.1%	28.7%	28.7%
Fiscal shocks	77.5%	66.2%	64.9%	64.8%

Main Takeaways from FEVD Findings

- Global shocks account for about one third of total variance of output growth on average across countries. The effects of global shocks on output is slightly lower, about a quarter, for short (Y=0) horizon.
- Global shocks are comparatively less important for the growth of debt-to-GDP ratio, about one eights for short horizon (Y=0) and one fifth of total variance at longer horizons.

Concluding Remarks

- We have revisited the relationship between the public debt and output growth. We do not provide any indication about the direction of causality between public debt and growth, and in fact we allow for causality to run both ways.

- There is no simple universal threshold above which debt-to-GDP becomes a significant brake on growth.

- There is a weak evidence of an interactive threshold effect.

- In the very long-run, debt and output must be cointegrated with a unit coefficient, but important departures can persist for protracted periods of time. We find no cointegration between the output and public debt levels for about half of the countries in our sample.

- In terms of short-run dynamics, we found that effects of global fiscal shocks are more effective in stimulating output than country-specific fiscal shocks, suggesting that coordinated fiscal actions are more effective.

Data I

- The CPI and real GDP data series are from the IMF *International Financial Statistics* database except for CPI data for Brazil, China and Tunisia which is from the IMF *World Economic Outlook* database and CPI data for UK which is from the Reinhart and Rogoff's *Growth in a Time of Debt* database.

- The gross government debt/GDP data series are from Reinhart and Rogoff (2011) and their most-up-to date *From Financial Crash to Debt Crisis* online database, except for Iran, Morocco, Nigeria, and Syria for which the IMF FAD *Historical Public Debt* database was used instead.

- We focus on gross debt data due to difficulty of collecting net debt data on a consistent basis over time and across countries. Moreover, we use public debt at the general government level for as many countries as possible.

Data II

- Since our analysis allows for slope heterogeneity across countries, we need a sufficient number of time periods to estimate country-specific coefficients. To this end, we include only countries in our sample for which we have at least 30 consecutive annual observations on debt, inflation and GDP.

- Subject to this requirement we ended up with 40 countries (covering most regions in the world and include advanced, emerging and developing countries).

- We also set the minimum cross section dimension to 20, since to take account of error cross sectional dependence we need to form cross section averages based on a sufficient number of units. We ended up with an unbalanced panel covering the sample period 1965-2010, with $T_{\text{min}} = 30$, and $N_{\text{min}} = 20$ across all countries and time periods.