

Opening Models of Asset Prices and Risk to Non-Routine Change*

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March 17, 2012

Paper revised for the Institute for New Economic Thinking (INET)
Plenary Conference
Berlin, April 12-15, 2012

*The authors are grateful to the Institute for New Economic Thinking for supporting the research on which this paper is based.

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Contents

1	Introduction	1
2	Irregular Swings in Asset Prices and Risk	4
3	Endogenous Prospect Theory of Risk	7
3.1	Individual Preferences	8
3.1.1	Original Utility Function	9
3.1.2	From Prospective to Expected Values	10
3.1.3	Limits to Speculation and an Individual Uncertainty Premium	11
3.2	Momentary Equilibrium Price and the Uncertainty Premium	13
3.3	Imperfect Knowledge and Expected Values	15
4	An IKE Gap Model of the Market Premium	15
4.1	The Gap Effect	16
4.2	Contingent Predictions of the Market Premium	18
4.3	The Gap Model and Momentary Equilibrium	19
5	Bubbles and Lost Fundamentals: Artifacts of the Contemporary Approach	19
5.1	Technical Trading and Psychology's Inability to Sustain Swings	20
5.1.1	Fundamental Underpinning of Psychological and other Non-Fundamental Considerations	21
5.2	Fundamentals Matter, But in Non-Routine Ways	23
6	An IKE Account of Asset Price Swings	26
6.1	An Individual's Price Forecast	27
6.2	Non-Routine Revisions of Forecasting Strategies	28
6.2.1	Formalization of Guardedly-Moderate Revisions	30
6.2.2	Psychological Motivation for Guardedly-Moderate Revisions?	32
6.3	Irregular Swings in an Individual's Price Forecast	33
6.4	Bulls, Bears, and Irregular Swings in the Aggregate Forecast	34
6.4.1	Moving to the Aggregate Level	34
6.5	Irregular Price Swings	36
6.5.1	Risk and Bounded Instability	37
7	Contingent Predictions of Long Swings and Their Compatibility With Rationality	39

8 An Intermediate View of Markets and the Role of the State

40

1 Introduction

Financial markets are, in most respects, prototypes of the markets for which much of contemporary economic analysis was designed. They are characterized by a large number of buyers and sellers; powerful monetary incentives; few, if any, barriers to entry and exit; no impediments to the adjustment of prices; and a plethora of available information that is quickly disseminated around the world. We would expect that financial markets would offer the best opportunity for contemporary economic models to provide explanations of market outcomes. But it is precisely in these markets that contemporary macroeconomic and finance theory have encountered many of their most glaring empirical difficulties.

In our companion paper for this conference, we trace contemporary economic theory's empirical and epistemological problems to how it models change in the causal process underpinning market outcomes.¹ In financial markets, outcomes are driven primarily by market participants' forecasts of prices and risk. As time passes, participants revise their forecasting strategies in ways that they themselves, let alone an economist, cannot fully foresee. Economic policies, institutions, the state of technology, and other features of the social context within which participants make decisions also change in novel ways. Thus, change in financial markets, and in capitalist economies more broadly, is, to a significant degree, non-routine, for it cannot be adequately represented in advance with mechanical rules and procedures. Yet, the hallmark of contemporary theory is the core premise that an economist can fully specify, in terms of some causal factors, how individuals alter the way that they make decisions, and how market outcomes unfold over time.

In this paper, we follow an alternative approach to economics analysis – called Imperfect Knowledge Economics (IKE) – and develop a model of asset prices and risk that has explicit mathematical microfoundations, and yet remains open to non-routine change.² At any given point in time, the general structure of our IKE model is no different from that of other macroeconomics and finance models. It consists of representations of individuals' preferences, forecasting behavior, constraints, and decision rule in terms of a set of causal (often called informational) variables, which portray the influence of economic policy, institutions, and other features of the social context. It also entails an aggregation rule and processes for the informational variables. The fact that participants revise their forecasting strategies, and that the social context changes, at least intermittently, implies that we will need different structures

¹See also Frydman and Goldberg (2007, 2011, 2013).

²The analysis in this paper builds on Frydman and Goldberg (2007, 2008).

– different specifications of individuals’ decision-making and the social context – in different time periods. Like other models, the adequacy of an IKE model in accounting for the time-series data hinges on how it characterizes this change.

Most macroeconomists and finance theorists use mechanical rules and procedures that fully prespecify any change in the structures of their models. Indeed, the vast majority of economists construct models that rely on the same structure to represent individual behavior and aggregate outcomes at every point in time. These time-invariant models presume that economic policy and the ways that individuals forecast market outcomes *never* change. Their key feature is that they represent outcomes in all time periods with a single conditional probability distribution.

To be sure, economists sometimes portray change in their models. However, when they do, they fully prespecify it with deterministic or probabilistic rules that suppose not only that individual decision-making or changes in the social context exhibit regularities, but that these regularities can be adequately characterized with mechanical rules and procedures. As a result, even if they allow for change, these models specify it fully in advance: conditional on the values of the causal variables at a point in time, they determine *exactly* all potential changes and the probabilities with which they might occur—in the past, present, and future all at once. Fully predetermined models represent change as random deviations from a fully predetermined time path, and thus assume away non-routine change altogether.

By sharp contrast, Knight (1921) and Keynes (1921, 1936) argued that most business decisions are fraught with “radical uncertainty,” a situation in which individual decisions cannot be adequately represented with a standard probability distribution. An extreme version of such uncertainty is often thought to force participants to act according to their “animal spirits,” psychological impulses that are largely disconnected from any fundamental considerations that might drive outcomes. However, unless participants’ decision-making could be connected, at least during some time periods, to the causal factors observable by economists, no formal economic theory with empirical content would be possible. As Phelps (2008) recently put it, “animal spirits can’t be modeled.”

At the same time, even if one supposes that economic behavior exhibits some regularities, non-routine change alters how market outcomes unfold over time in ways that put an overarching probabilistic representation of outcomes out of reach of economic analysis. IKE, therefore, stakes out an intermediate position between radical uncertainty, which, in its extreme, “animal spirits” version, denies that economists can formu-

late testable mathematical models of any features of the causal process driving change, and the contemporary presumption that a single conditional probability distribution can adequately represent this process. Although IKE jettisons the presumption that individual decision-making and changes in the social context exhibit regularities that conform to fully prespecified rules, it explores the possibility that such behavior nonetheless exhibits regularities. But, at best, we would expect these regularities to be context-dependent and qualitative. We would also expect them to be contingent: they begin and cease to be relevant at moments that no one can fully foresee.

In modeling asset prices and risk, an economist's assumptions concerning market participants' forecasting behavior play a crucial role. In searching for empirically relevant regularities that might characterize such behavior, we are guided by empirical findings from behavioral economics and other social sciences. For example, to portray how market participants sometimes revise their forecasting strategies, we make use of psychological findings that indicate that individuals are often slow to revise their beliefs in the face of new evidence. We also build on Keynes's (1936) insight that social conventions are important for understanding how participants form their expectations in financial markets. But our IKE model formalizes these insights with qualitative and contingent conditions, rather than with the mechanical rules that behavioral-finance models use to represent behavior.

We also rely on behavioral findings in modeling market participants' risk preferences, drawing on Kahneman's and Tversky's (1979) prospect theory. But we use an extension of their original formulation, which we call endogenous prospect theory, that recognizes that outcomes cannot be represented with an overarching probability distribution.³

We show that opening a mathematical model to non-routine change and imperfect knowledge on the part of economists enables us to incorporate both fundamental variables (such as earnings and interest rates), on which rational expectations (REH) theorists focus, and psychological and social considerations (such as confidence and conventions), which behavioral economists emphasize, without presuming obvious irrationality on the part of market participants. We also show that, despite its openness to economists' ever-imperfect knowledge about the process driving

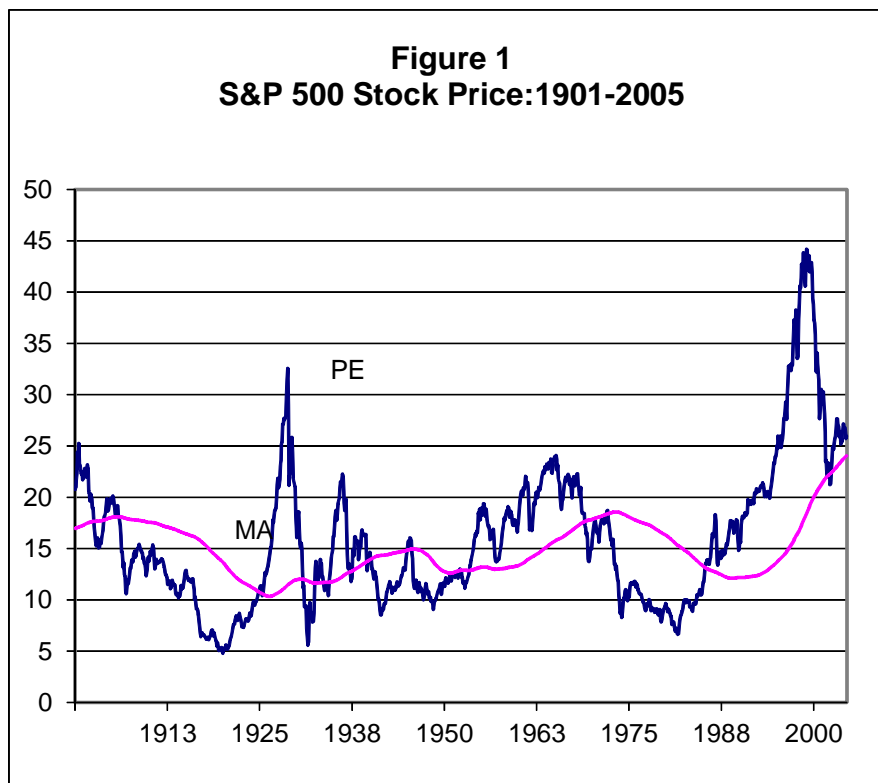
³See Frydman and Goldberg (2007). Kahneman and Tversky developed their theory for experimental conditions that assume away imperfect knowledge on the part of economists. Subsequent applications of the theory were implemented in fully predetermined models. Endogenous prospect theory extends the applications of the theory to IKE models. It also solves several problems in applying prospect theory to modeling asset markets.

change, the model generates implications that can be confronted with time series evidence.

2 Irregular Swings in Asset Prices and Risk

In this paper, we model market outcomes that contemporary macroeconomics and finance theory have found difficult to explain. We focus on the tendency of asset prices to undergo prolonged swings away from and toward estimates of common benchmark levels and the two-way interdependence of this behavior and financial risk.

Two examples of price fluctuations in asset markets are provided in figures 1 and 2, which plot the Standard and Poor's 500 price index relative to underlying earnings and the British pound-U.S. dollar (BP/\$)



exchange rate, respectively, along with a common benchmark.⁴ The figures show that asset prices can move away from benchmark levels for

⁴The price-earnings (PE) ratio in figure 1 is based on a trailing 10-year moving average of earnings. The data are from Shiller (2000), which are updated on his web site: www.econ.yale/~shiller. The benchmark in the figure is a 20-year moving average of Shiller's PE ratio. The benchmark exchange rate in figure 2 is a purchasing power parity (PPP) rate, which was calculated using the *Big Mac* PPP exchange rate

years at a time. But the instability is bounded: eventually these swings undergo sustained countermovements back toward benchmark values. If prices happen to reach these values, they often overshoot them and begin trending away from the other side. Moreover, although such fluctuations are a recurrent feature of market outcomes, the observed swings are irregular: the magnitude and duration of upswings and downswings vary from one episode to the next in ways that do not seem to follow any consistent pattern. The inability of REH-based models to account for these features of asset price fluctuations is well known.⁵

The other aggregate regularity that we model involves the market risk premium – the anticipated excess return that market participants in the aggregate require in order to hold the available supply of the risky asset. REH risk premium models relate this premium to the second-moment properties of the data. However, ever since Mehra and Prescott (1985), economists have known that REH models are grossly inconsistent with time-series data on the market risk premium.

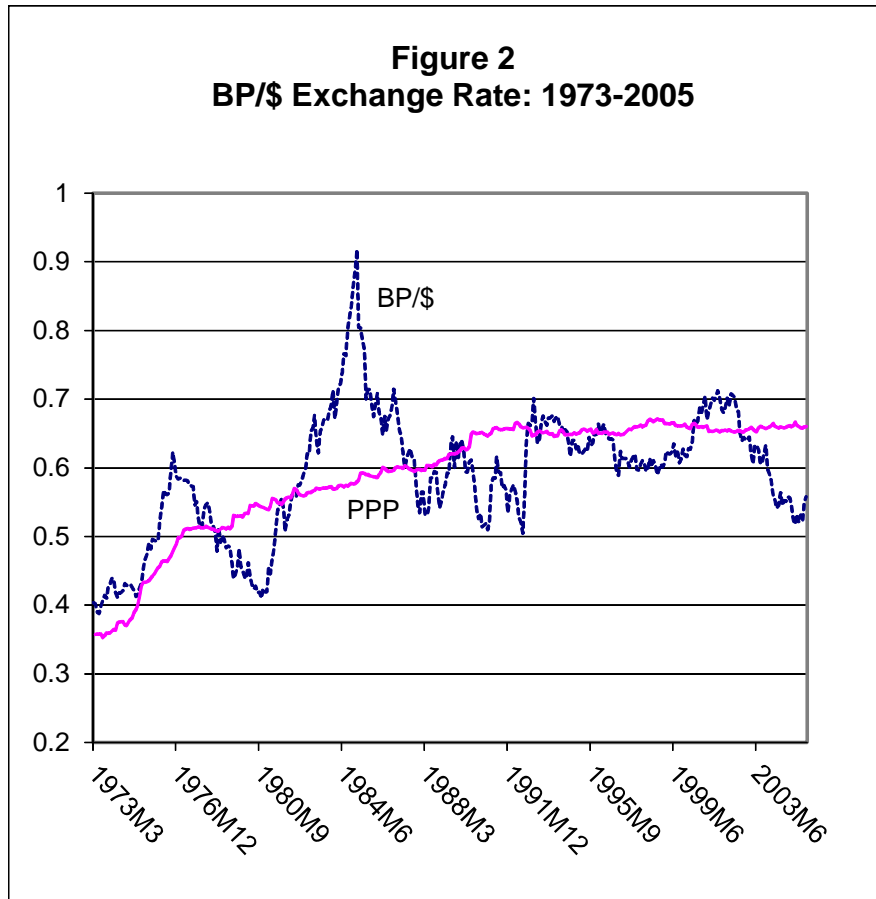
There is much evidence that the market premium also undergoes swings and that these fluctuations depend on how the asset price moves relative to benchmark values: as participants bid the price farther away from estimates of benchmark values, the riskier it becomes to gamble on an even greater gap. Such behavior can be seen in figure 3, which plots the *ex ante* excess return on holding U.S. dollar long positions in the BP/\$ market and the gap between the exchange rate and its PPP value.⁶ The tendency for the market premium to move positively with the swing in the exchange rate relative to PPP is striking.

reported in the April 6, 1990 issue of *The Economist* magazine (which was BP1.96/\$1) and CPI-inflation-rate differentials from the IMF's *International Financial Statistics*. Historical averages of PE ratios and PPP exchange rates have long traditions as benchmark values in stock and currency markets, respectively.

⁵For stock and currency markets, see Shiller (2003) and Frydman and Goldberg (2007), respectively, and references therein.

⁶We use survey data on exchange rate forecasts to measure the market premium and the *Big Mac* PPP exchange rate as in figure 2. The survey data are from Money Market Services International (MMSI), which entail participants' median responses concerning their four-week ahead point forecast of the exchange rate. For more details concerning the time plots in figure 3, see Frydman and Goldberg (2007, chapter 12). Other studies that have used MMSI's survey data include Frankel and Froot (1987) and Froot and Frankel (1989).

Figure 2
BP/\$ Exchange Rate: 1973-2005

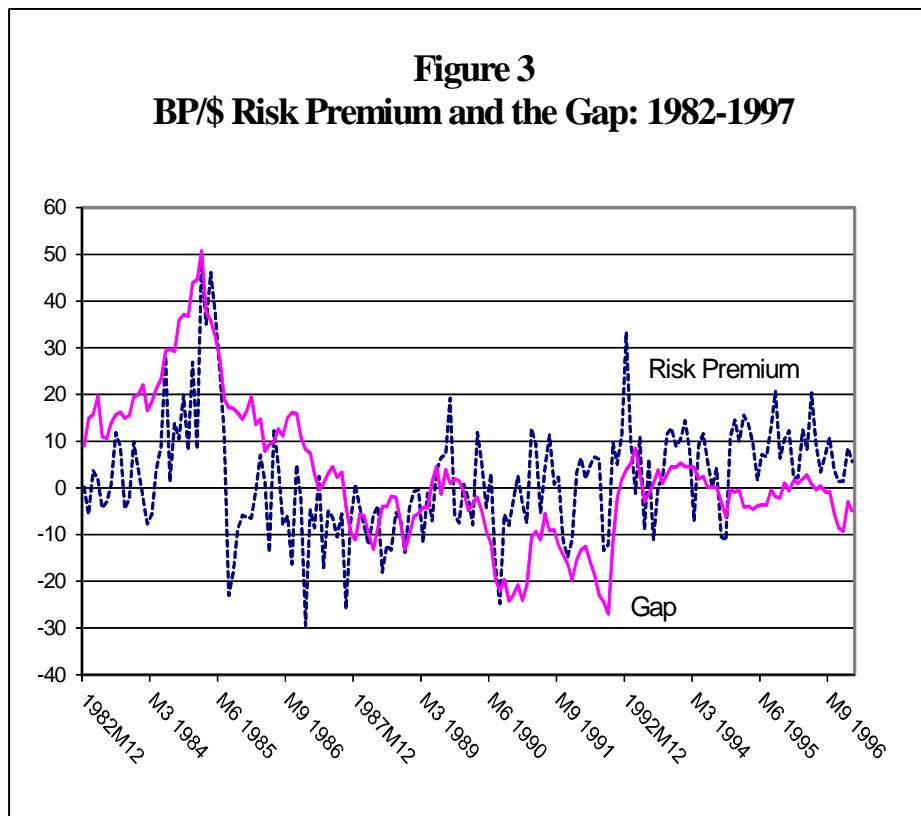


Frydman and Goldberg (2007) and Stillwagon (2010) undertake more formal statistical analysis using parametric and nonparametric procedures and find a positive relationship between the premium and the gap in the three largest currency markets.

As for other asset markets, there is widespread understanding in policy making circles that risks grow as upswings in equity and housing prices continue for prolonged periods of time.⁷ As the financial crisis that began in 2007 dramatically showed, long upswings in these prices are eventually followed by sharp and protracted downswings. Zheng (2009), for example, uses regression analysis to examine the relationship between the premium on stocks over bonds and the gap from benchmark values. Her results are broadly consistent with those for currency markets: the equity premium tends to rise and fall in concert with swings in equity prices relative to the benchmark.

⁷For example, see Borio (2003), Borio and Shim (2007), and BIS (2010). Reinhart and Rogoff (2009) look at data going back as far as eight centuries for 66 countries spanning six continents and find that excessive movements in real exchange rates, real housing prices, and real stock prices are among the top five predictors of subsequent sharp reversals and crises.

In the remainder of this paper, we develop microfoundations for an IKE model that can account for the long-swings nature of fluctuations in asset prices and risk – and for the connection between the two – without presuming that individuals forego obvious profit opportunities.



3 Endogenous Prospect Theory of Risk

We portray individual decision-making with the usual assumption of utility maximization. But, in specifying an individual’s preferences and decision rule under uncertainty, we make use of endogenous prospect theory, rather than a standard risk-averse specification and expected utility theory.⁸ This alternative specification of preferences and decision-making, together with our IKE representation of forecasting, leads to a new model of risk in asset markets: individuals’ assessments of the riskiness of holding speculative positions depend on the gap between the

⁸There is much experimental evidence showing that standard specifications are grossly inconsistent with how individuals actually behave. See Kahneman and Tversky (1979) and Barberis and Thaler (2003), and references therein.

asset price and their perceptions of common benchmark levels, rather than on standard measures of volatility as is usually the case.

3.1 Individual Preferences

The structure of our model assumes that individuals hold their non-monetary wealth in a risky and a riskless asset, call them stocks and bonds, respectively.⁹ Individuals can issue bonds and take short positions in stocks without limit, although in equilibrium they must, in the aggregate, hold the available supplies of stocks and outside bonds.

We denote the *ex post* nominal return on holding a long position in stocks for one period by R_{t+1} and express it using a logarithmic approximation:

$$R_{t+1} = P_{t+1} - P_t - i_t \quad (1)$$

where P_t denotes the log level of the time- t stock price, i_t is the riskless nominal return on bonds, and, for the sake of simplicity, we ignore dividends.¹⁰ Analogously, the one-period return on a short position in stocks is given by $-R_{t+1}$.

An individual's preferences over alternative long and short positions in stocks are specified in terms of gains and losses in wealth relative to some reference level, which Kahneman and Tversky call "reference dependence." The one-period change in wealth can be written as follows:

$$\begin{aligned} \Delta W_{t+1}^i &= W_{t+1}^i - \Gamma_t^i \\ &= W_t^i [a_t^i R_{t+1} + (1 + i_t^A - \pi_t)] - \Gamma_t^i \end{aligned} \quad (2)$$

where W_t^i denotes an individual i 's non-monetary real wealth at time t , a_t^i denotes the share of stocks in her portfolio at time t ($a_t^i < 0$ implies a short position), π_t is the non-stochastic rate of inflation, Γ_t^i denotes an individual's reference level, and Δ is the first-difference operator.¹¹ Whenever $\Delta W_{t+1}^i > 0$ ($\Delta W_{t+1}^i < 0$), an individual is said to experience a gain (a loss).

⁹In Frydman and Goldberg (2007), we develop the model in the context of currency markets.

¹⁰Alternatively, we could redefine P_t to include dividends without changing the main conclusions of our analysis.

¹¹The real returns on stocks and bonds are $P_{t+1} - P_t + i_t - \pi_t$ and $i_t - \pi_t$, respectively. As such, next-period's wealth is $W_{t+1}^i = W_t^i [1 + a_t^i (S_{t+1} - S_t + i_t^B - \pi_t) + (1 - a_t^i) (i_t^A - \pi_t)] = W_t^i [a_t^i R_{t+1} + (1 + i_t^A - \pi_t)]$. The assumption of a non-stochastic inflation rate is common in the portfolio-balance literature because asset prices are considerably more volatile than goods prices. See Krugman (1981), Frankel (1982), and Dornbusch (1983).

In the context of financial markets, it is natural to set an individual’s reference point to the level of wealth she would obtain were she to stay out of the stock market completely:¹²

$$\Gamma_t^i = W_t^i (1 + i_t^A - \pi_t) \quad (3)$$

Substitution of (3) into (2) yields the following expression for the change in an individual’s wealth relative to her reference level:

$$\Delta W_{t+1}^i = a_t^i W_t^i R_{t+1} \quad (4)$$

A positive realization of R_{t+1} , which we denote by r_{t+1}^+ , leads to a gain for an individual who holds a long position (that is, $a_t^i > 0$) and a loss if she holds a short position (that is, $a_t^i < 0$). A negative realization of R_{t+1} , which we denote by r_{t+1}^- , leads to the converse. It is common to refer to individuals who hold long and short positions as bulls and bears, respectively.

3.1.1 Original Utility Function

In addition to reference dependence, Kahneman and Tversky’s experimental findings imply that individuals are “loss averse” – their disutility from losses substantially exceeds the utility from gains of the same magnitude. Moreover, their preferences display “diminishing sensitivity”: their marginal utility of both gains and losses decreases with their size.¹³ Tversky and Kahneman (1992) propose a utility function that embodies these characteristics, which we express as follows:

$$V(\Delta W) = \begin{cases} (W |ar^g|)^\alpha \\ -\lambda(W |ar^l|)^\beta \end{cases} \quad (5)$$

where λ is a constant and r^g and r^l denote a gain and loss, respectively, on an open position in stocks. It is convenient to define gains on *both* long and short positions as positive values and losses as negative values:

¹²In general, however, each individual chooses her own reference level and neither prospect theory nor available experimental evidence provides guidance as to how an economist should represent it. Other studies that use this reference level in applying prospect theory to modeling asset prices include Barberis, Huang, and Santos (2001) and Barberis and Huang (2002). See Ang, Bekaert, and Liu (2004) for an extensive discussion of the difficulties inherent in modeling the reference level in the context of prospect theory.

¹³Diminishing sensitivity implies the familiar concavity of the utility function in the domain of gains. However, in the domain of losses, the utility function is convex, implying that individuals have a greater willingness to gamble as the magnitude of their potential loss rises.

$$\begin{aligned} r^g &= r_{t+1}^+ \text{ and } r^l = r_{t+1}^- && \text{if } a > 0 \\ r^g &= -r_{t+1}^- \text{ and } r^l = -r_{t+1}^+ && \text{if } a < 0 \end{aligned} \quad (6)$$

The utility function in (5) implies that the degree of loss aversion – defined as the ratio of the disutility of losses over the utility of gains of the same magnitude – depends on the size of an individual’s stake in the game, a :¹⁴

$$\Lambda = \lambda(|ar^l|)^{\beta-\alpha} \quad (7)$$

Like expected utility theory, prospect theory portrays an individual’s decision-making by assuming that she maximizes a weighted sum of the utilities of single outcomes, called "prospective utility." In the context of our model, an individual chooses a_t^i so as to maximize:

$$PU_t^i = |a_t^i W_t^i|^\alpha \sum_k^{K^+} \omega_{t,k}^i \left(\hat{r}_{t+1,k}^g \right)^\alpha - \lambda |a_t^i W_t^i|^\beta \sum_k^{K^-} \omega_{t,k}^i \left(-\hat{r}_{t+1,k}^l \right)^\beta \quad (8)$$

where the $\hat{r}_{t+1,k}^g$ ’s and $\hat{r}_{t+1,k}^l$ ’s are the single gains and losses that an individual forecasts are possible, respectively, K^+ and K^- denote the numbers of these single outcomes, respectively, and the $\omega_{t,k}^i$ ’s are an individual’s decision weights.¹⁵

3.1.2 From Prospective to Expected Values

In order to derive implications from models based on prospect theory and confront them with time-series data, we must relate individuals’ prospective utilities to observable causal variables. Behavioral-finance economists address this problem by equating decision weights with probabilities and setting $\alpha = \beta = 1$, thereby assuming that the utility function is linear over potential gains and losses and enabling them to express prospective utility in terms of expected values.¹⁶

In the context of our model, these assumptions would imply:

$$PU_t^i = |a_t^i W_t^i| \left[\hat{r}_{t|t+1}^{i,j} - (1 - \lambda) \hat{l}_{t|t+1}^{i,j} \right] \quad (9)$$

where $\hat{r}_{t|t+1}^{i,j} = E_t^i[R_{t+1}|Z_t^i] \geq 0$ denotes a bull’s ($j=L$) or bear’s ($j=S$) point forecast of the excess return on holding an open position for one

¹⁴This can be seen by rewriting (5) as $V(\Delta W) = (W|ar^g|)^\alpha - \lambda(W|ar^l|)^{\beta-\alpha}(W|ar^l|)^\alpha$.

¹⁵We follow Tversky and Kahneman (1992) in assuming a finite set of outcomes.

¹⁶For example, see Barberis *et al* (2001).

period, which is implied by her forecasting strategy (which we represent by a conditional probability distribution, $\mathcal{P}_t^i(R_{t+1})$) and the values of the informational variables a time t , Z_t^i . An individual's $\hat{l}_{t|t+1}^{i,j}$ is the conditional expected value of the “loss part” of her distribution of R_{t+1} , which for a bull involves the negative realizations of R_{t+1} ,

$$\hat{l}_{t|t+1}^{i,L} = E_t^i[R_{t+1}^L < 0 | Z_t^i] < 0$$

whereas for a bear, it involves the positive realizations,

$$\hat{l}_{t|t+1}^{i,S} = E_t^i[R_{t+1}^S < 0 | Z_t^i] < 0 \quad (10)$$

where $R_{t+1}^L = P_{t+1} - P_t - i_t$ and $R_{t+1}^S = i_t + P_t - P_{t+1}$ denote returns on a long and short position, respectively. We refer to $\hat{l}_{t|t+1}^{i,j}$ as an individual's expected potential loss on a unit position.

The specification in (9) implies a risk premium that depends on an individual's degree of loss aversion and her forecast of the potential loss, $(1 - \lambda) \hat{l}_{t|t+1}^{i,j} > 0$.¹⁷ As long as her expected excess return, $\hat{r}_{t|t+1}^{i,j}$, exceeded her risk premium, she would want to take an open position in stocks. However, because (9) is linear in the decision variable, a_t^i , she would want to take a speculative position of unlimited size. As it stands, a well-defined equilibrium in which individuals hold diverse forecasts is ruled out.¹⁸

3.1.3 Limits to Speculation and an Individual Uncertainty Premium

In Frydman and Goldberg (2007), we show that Tversky and Kahneman's utility function implies limits to speculation if one recognizes diminishing sensitivity and sets $\beta > \alpha > 0$. The expression in (7) shows that under this assumption, an individual's loss aversion is endogenous, rising as the size of her open position rises and implying concavity of PU_t^i in a_t^i .¹⁹ However, the non-linearity creates several problems, including

¹⁷We recall that $\lambda > 1$ and $\hat{l}_{t|t+1}^{i,j} < 0$.

¹⁸Modeling finite speculative positions, widely called “limits to arbitrage,” is viewed as one of the two main pillars of behavioral finance (see Barberis and Thaler, 2003). Linearizing Tversky and Kahneman's utility function has led some economists to model finite speculative positions by relying on the assumptions of risk aversion and loss aversion in specifying preferences. The reliance on risk aversion is puzzling, however, given that its rejection in favor of loss aversion has come to be viewed as one of behavioral economics' key findings. For a review article on modeling limits to arbitrage, see Gromb and Vayanos (2010).

¹⁹We are unaware of any experimental evidence that directly demonstrates a positive relationship between an individual's degree of loss aversion and the size of her stake. But as Myron Scholes emphasized in remarks at a conference on “Derivatives

an inability to express the aggregate demand for stocks, $a_t W_t$, in terms of aggregate forecasts, $\hat{r}_{t|t+1}$ and $\hat{l}_{t|t+1}$.²⁰

Endogenous prospect theory addresses these problems by reformulating the utility function in (5) as follows:

$$V(\Delta W) = \begin{cases} (W|a|)^\alpha |r^g| \\ -\lambda_1 (W|a|)^\alpha |r^l| - \frac{\lambda_2}{-\hat{l}_{t|t+1}^{i,j}} (W|a|)^{\alpha+1} |r^l| \end{cases} \quad (11)$$

where $\lambda_1 > 1$ and $\lambda_2 > 0$ ensure loss aversion. We show in Frydman and Goldberg (2007) that the specification in (11) is consistent with all of Kahneman and Tversky's (1979) main experimental findings.²¹ And because it implies endogenous loss aversion, it implies limits to speculation.²²

To portray an individual's portfolio decision at time t , we maximize the prospective utility function in (11) with respect to a_t^i , taking as given the individual's point forecasts of the excess return and potential unit loss on open positions in stocks. This yields the following optimal long or short position, depending on whether the individual is a bull or bear:

$$a_t^{i,L} W_t^i = \frac{\alpha}{\lambda_2(\alpha + 1)} \left[\hat{r}_{t|t+1}^{i,L} - (1 - \lambda_1) \hat{l}_{t|t+1}^{i,L} \right] \quad (12)$$

$$-a_t^{i,S} W_t^i = \frac{\alpha}{\lambda_2(\alpha + 1)} \left[\hat{r}_{t|t+1}^{i,S} - (1 - \lambda_1) \hat{l}_{t|t+1}^{i,S} \right] \quad (13)$$

where $a_t^{i,L}$ and $a_t^{i,S}$ are constrained to be nonnegative and nonpositive, respectively,²³ and we ignore differences in all preference parameters.²⁴

2003: Reports from the Frontiers," held at the Stern School of Business, January 2003, such a relationship accords well with how participants in real-world markets actually behave.

²⁰ Another problem with the specification in (5) when $\beta > \alpha > 0$ is that for small positions sizes, it violates the basic assumption of loss aversion.

²¹ This specification implies what we call "endogenous sensitivity": the marginal value of both gains and losses decreases with position size (as it does in Tversky and Kahneman's original specification), except when the size of the stake becomes large, at which point the marginal value of losses increases with position size. Although Tversky and Kahneman (1992) assume diminishing sensitivity throughout the domain of gains and losses, they recognize in Kahneman and Tversky (1979) that this behavior is typical for smaller gambles, and that increasing sensitivity may characterize individual preferences for larger gambles.

²² The degree of loss aversion is now given by $\Lambda = \lambda_1 + \frac{\lambda_2}{-\hat{l}_{t|t+1}^{i,j}} W|a|$.

²³ The inequality constraints on $a_t^{i,L}$ and $a_t^{i,S}$ reflect the possibility that an individual's optimal position size in stocks would be zero if the her expected excess return did not outweigh her concern about potential losses, as represented by the expected unit loss, $(1 - \lambda_1) \hat{l}_{t|t+1}^{i,L}$.

²⁴ We consider an IKE model of currency returns with heterogeneity over preferences in Frydman and Goldebrg (2007).

An individual's "flow" demand for or supply of stock (that is, the value of stock that she wishes to buy or sell) at time t is thus given by:

$$D_t^i = a_t^{i,j} W_t^i - S_t^i \quad (14)$$

where S_t^i denotes the value of stock with which the individual enters period t . A $S_t^i < 0$ implies that the individual holds a short position entering the period, while $D_t^i < 0$ indicates that she is a seller of stocks at t .²⁵

The expressions in (12) and (13) show that endogenous prospect theory leads to a new specification for the minimum expected return that individuals require in order to take risky positions in stocks, which we call an individual "uncertainty premium".²⁶

$$\widehat{up}_{t+1}^{i,j} = (1 - \lambda_1) \widehat{l}_{t|t+1}^{i,j} > 0 \quad (15)$$

An individual's uncertainty premium depends on her forecast of the potential loss from speculating, rather than on standard measures of volatility.

3.2 Momentary Equilibrium Price and the Uncertainty Premium

Endogenous prospect theory and portfolio balance lead to a new *momentary* equilibrium condition for the stock market. It is obtained by aggregating individuals' demands and supplies in (14) using wealth shares and assuming that the stock price adjusts instantaneously to balance the total buying and selling in the market at every point in time:

$$\widehat{r}_{t|t+1} = \widehat{up}_{t|t+1} + \lambda_2 \frac{S_t}{W_t} \quad (16)$$

where $\widehat{r}_{t|t+1} = \widehat{P}_{t|t+1} - P_t - i_t$, $\widehat{P}_{t|t+1}$ represents the aggregate of participants' conditional point forecasts of P_{t+1} , S_t and W_t are the available supplies of stock and total nonmonetary wealth held by market participants, respectively, and $\widehat{up}_{t|t+1}$ is the aggregate uncertainty premium,

$$\widehat{up}_{t|t+1} = \widehat{up}_{t|t+1}^L - \widehat{up}_{t|t+1}^S = \frac{1}{2} (1 - \lambda_1) \left(\widehat{l}_{t|t+1}^L - \widehat{l}_{t|t+1}^S \right) \quad (17)$$

²⁵ Although an individual may remain a bear from one period to the next, her trading may nonetheless add to buying in the market. This would be the case if she wanted to reduce the size of her short position, that is, if $0 > a_t^{i,s} W_t^i > S_t^i$, so that $D_t^i > 0$. Analogously, a bull's trading would add to selling in the market if $0 < a_t^{i,l} W_t^i < S_t^i$, so that $D_t^i < 0$.

²⁶ We refer to this minimum return as an uncertainty premium to highlight Knight's (1921) distinction between uncertainty and risk, which recognizes that the risk in markets stems from the inherent imperfection of knowledge.

which depends on the uncertainty premium of the bulls minus the uncertainty premium of the bears.²⁷

Equation (16) defines momentary equilibrium in the stock market as the situation in which the expected excess return exceeds the uncertainty premium in the aggregate sufficiently for market participants to willingly hold the available supply of stocks (and bonds). The implied market premium $-\widehat{pr}_{t|t+1} = \widehat{up}_{t|t+1} + \lambda_2 \frac{S_t}{W_t}$ depends on both the aggregate uncertainty premium and relative asset supplies. This specification shows that in order to account for one of the key features of fluctuations of $\widehat{pr}_{t|t+1}$ in asset markets – reversals in its algebraic sign from one time period to another (see figure 3) – we must recognize the coexistence of both bulls and bears in the market.²⁸

Endogenous prospect theory and portfolio balance imply that the unfolding of the equilibrium price, P_t , depends on movements in participants' point forecasts of the next-period's price and potential unit loss, as well as the interest rate on bonds and relative asset supplies. However, to simplify our discussion, we assume that i_t and $\frac{S_t}{W_t}$ are constant and set $i + \lambda_2 \frac{S}{W} = 0$, yielding the following equilibrium condition:

$$\widehat{P}_{t|t+1} - P_t = (1 - \lambda_1) \widehat{l}_{t|t+1} \quad (18)$$

where $\widehat{l}_{t|t+1} = \frac{1}{2} \left(\widehat{l}_{t|t+1}^L - \widehat{l}_{t|t+1}^S \right)$. This simplification enables us to focus on the main drivers of price – movements in individuals' forecasts, $\widehat{P}_{t|t+1}$ and $\widehat{l}_{t|t+1}$.²⁹

²⁷In general, the wealth shares of the bulls and bears vary over time. However, we abstract from such variation and assume that the wealth share of the bulls, ω^L , and that of the bears, ω^S , are constant and equal to a half. More generally, $\widehat{up}_{t|t+1} = (1 - \lambda_1) \left(\omega^L \widehat{l}_{t|t+1}^L - \omega^S \widehat{l}_{t|t+1}^S \right)$.

²⁸Sign reversals of $\widehat{pr}_{t|t+1}$ occur when the relative uncertainty premiums of the bulls and bears pendulate sufficiently to offset the influence of $\lambda_2 \frac{S_t}{W_t} > 0$. By sharp contrast, REH risk premium models are unable to account for such behavior. See Mark and Wu (1998) and Zheng (2009), which find that consumption capital asset pricing models are grossly inconsistent with sign reversals of $\widehat{pr}_{t|t+1}$ in currency and stock markets, respectively. Frydman and Goldberg (2007) examine the inability of the portfolio-balance approach sketched here under the assumptions of expected utility theory and risk aversion to explain sign reversals in currency markets, regardless of whether we assume diversity of individuals' forecasting strategies.

²⁹The variation in interest rates and relative asset supplies are an order of magnitude or more lower than the variation in stock and currency prices. Consequently, the variation of R_{t+1} and $P_{t+1} - P_t$ are of roughly equal magnitude.

3.3 Imperfect Knowledge and Expected Values

As time advances, news becomes available about causal variables and an individual may decide to revise her forecasting strategy, which we represent with a new conditional probability distribution, $\mathcal{P}_{t+1}^i(R_{t+2})$. Both news and a revised forecasting strategy will, in general, lead her to alter her point forecast not only of price, but also of the potential unit loss, and thus of the riskiness, from open positions in stocks. We would not expect such revisions to follow any mechanical rule. But, how an individual's $\hat{r}_{t|t+1}^{i,j}$ and $\hat{l}_{t|t+1}^{i,j}$ change from one time period to the next may nonetheless exhibit qualitative and contingent regularities. To model this change, therefore, we make use of qualitative and contingent constraints that only partially prespecify it.

This IKE approach enables endogenous prospect theory's specification of risk to address a key concern that other asset-market studies applying prospect theory ignore: "[in] the typical situation of choice, where the probabilities of outcomes are not explicitly given[,]...the decision weights may be affected by other considerations, such as ambiguity or vagueness" (Kahneman and Tversky, 1979, pp. 288-289), which cannot be represented adequately with specific parametric functions of probabilities. These and other considerations arise from the imperfection of knowledge and are what distinguishes decision-making in real-world markets from that in experimental settings, where the experimenter fixes the fully prespecified probability distribution that governs subject's pay-offs.

Disregarding the distinction between decision-weighted sums and expected values of outcomes may be unavoidable in applying endogenous prospect theory. However, partially prespecifying change in $\hat{r}_{t|t+1}^{i,j}$ and $\hat{l}_{t|t+1}^{i,j}$ opens the model to the importance of ambiguity and vagueness, and, more generally, to ever-imperfect knowledge in individual decision-making.

4 An IKE Gap Model of the Market Premium

We turn first to portraying individuals' forecasts of the potential unit loss from speculating with qualitative and contingent constraints and show that the model generates predictions concerning the market premium that, although contingent, can be confronted with time series evidence. We sketch in section 6 how these predictions help explain why protracted price swings away from market participants' estimates of benchmark values are ultimately bounded.

We are guided not only by the empirical record on excess returns that we discussed in section 2, but by Keynes's (1936) account of asset

markets. Keynes argued that speculators are aware of asset prices' tendency to undergo irregular swings around benchmark levels, and that they take account of this feature of the social context in their attempts to forecast market outcomes. In discussing why an individual might hold cash rather than risky interest-bearing bonds, he observed that "what matters is not the absolute level of [the interest rate] r but the degree of its divergence from what is considered a fairly safe [benchmark] level of r , having regard to those calculations of probability which are being relied on" (Keynes, 1936, p.201).³⁰

Keynes's discussion of the importance of benchmark levels as anchors for asset price swings suggests that market participants look to the gap between the asset price and its benchmark value in forecasting the potential unit loss from speculation. As he put it, "Unless reasons are believed to exist why future experience will be very different from past experience, a...rate of interest [much lower than the safe rate], leaves more to fear than to hope, and offers, at the same time, a running yield which is only sufficient to offset a very small measure of fear [of capital loss]" (Keynes, 1936, p.202).

This insight leads us to relate an individual's expected unit loss on open positions in stocks to her assessment of the gap between the stock price and her perception of the benchmark value:

$$\widehat{l}_{t|t+1}^{i,j} = \widehat{l}_{t|t+1}^{i,j} \left(\widehat{gap}_t^i \right) \quad (19)$$

where $\widehat{gap}_t^i = P_t - \widehat{P}_t^{i,BM}$ and $\widehat{P}_t^{i,BM}$ denotes an individual's assessment at t of the benchmark stock price.^{31,32}

4.1 The Gap Effect

Over time, movements in \widehat{gap}_t^i lead an individual to alter her forecast of the potential unit loss, which may involve revisions of her forecasting strategy, $\mathcal{P}_t^i(R_{t+1})$. But, no matter how she might revise her strategy,

³⁰A benchmark level is, of course, specific to each asset market. Every individual arrives at her own determination of the benchmark value and so, in general, these assessments will differ across individuals. How individuals come to decide on a benchmark level is an open question. Keynes suggests in his discussion that conventions and the historical record play an important role.

³¹The gap could also be defined in terms of an individual's forecast of next-period's asset price rather the asset price itself, or some weighted average of the two, without affecting the conclusions of our analysis. See Frydman and Goldberg (2007).

³²Depending on the context, an individual's forecast of the potential unit loss may also depend on factors other than the gap. In Frydman and Goldberg (2007), for example, we include current account imbalances as an additional variable in modeling currency risk.

we suppose that movements of $\widehat{l}_{t|t+1}^{i,j}$ are characterized by a “gap effect” that depends on which side of the market she takes.

Consider, for example, an upswing in stock prices that has already climbed above participants’ assessments of benchmark levels. A bull forecasts that the price swing will continue, while a bear forecasts the opposite. Nonetheless, both contemplate the potential loss that they would incur if price were to move against them. If, for example, bulls were to increase their long positions and bid up price even more, they would also raise their assessments of the potential loss of being wrong: the greater the gap from their estimate of the benchmark, the more concerned they would tend to be about a reversal. Bears, on the other hand, would respond in the opposite way to a further rise in prices: they would tend to become more confident about an eventual reversal, and thus lower their assessments of the potential loss from their short positions.

Of course, how a market participant interprets the gap from benchmark values in assessing risk changes over time in ways that neither she nor an economist can specify fully in advance. Indeed, we present evidence in Frydman and Goldberg (2007, 2011) that the importance individuals attach to the gap when it is historically large is much greater than when it is historically small. No one can fully foresee the thresholds above or below which individuals might consider the magnitude of the gap to be large or small or how the crossing of these thresholds might impact their $\widehat{l}_{t|t+1}^{i,j}$ ’s.

Consequently, we formalize the influence of movements of \widehat{gap}_t^i on $\widehat{l}_{t|t+1}^{i,j}$ with qualitative restrictions:³³

$$\frac{\Delta \widehat{l}_{t|t+1}^{i,L}}{\Delta \widehat{gap}_t^i} < 0 \quad \text{and} \quad \frac{\Delta \widehat{l}_{t|t+1}^{i,S}}{\Delta \widehat{gap}_t^i} > 0 \quad (20)$$

These conditions allow for myriad possible non-routine revisions of an individual’s forecasting strategy between successive points in time as \widehat{gap}_t^i changes. They are thus consistent with myriad possible post-change conditional probability distributions, that is, with many $\mathcal{P}_{t+1}^i(R_{t+2})$ ’s. But, the conditions in (19) constrain the set of possible $\mathcal{P}_{t+1}^i(R_{t+2})$ ’s to share a common qualitative feature: they all imply that if the gap were higher at $t + 1$, a bull would forecast a larger potential unit loss (in size) from holding a long position, whereas a bear would forecast a smaller potential unit loss from holding a short position.

³³The less-than and greater-than inequalities that are used in specifying the gap conditions for a bull and bear follow from defining $\widehat{l}_{t|t+1}^{i,L}$ and $\widehat{l}_{t|t+1}^{i,S}$ as negative values. As such, greater (smaller) losses imply a fall (rise) in $\widehat{l}_{t|t+1}^{i,j}$.

The model does not pick out any one of these post-change distributions, and so it does not predetermine the exact size of the gap effect over any period. We thus refer to the post-change distributions that are consistent with the gap conditions as “partially predetermined.”³⁴ Any of these partially predetermined $P_{t+1}^i(R_{t+2})$ represents an individual’s forecasting strategy at $t + 1$.

4.2 Contingent Predictions of the Market Premium

Restricting change on the individual level with qualitative restrictions implies that our IKE model generates only qualitative predictions on the aggregate level. Equation (19) and the gap conditions in (20) imply a partially predetermined specification for the market premium, which we express as follows:

$$\widehat{pr}_{t|t+1} = \sigma_t \left(P_t - \widehat{P}_t^{\text{BM}} \right) \quad (21)$$

where $\sigma_t > 0$, $\widehat{P}_t^{\text{BM}}$ is the aggregate of individuals’ estimates of the benchmark and

$$\frac{\Delta \widehat{pr}_{t|t+1}}{\Delta \widehat{gap}_t} > 0 \quad (22)$$

Like on the individual level, the specification in (21) and (22) is consistent with myriad possible non-routine ways that the market premium might change from one point in time to the next; there are many post-change conditional probability distributions on the aggregate level, $\mathcal{P}_{t+1}(R_{t+2})$, at each point in time. However, each one of these partially predetermined distributions is characterized by an aggregate gap effect: if the aggregate gap were higher at $t + 1$, it would be associated with a higher market premium.

Consequently, our IKE gap model predicts that $\widehat{pr}_{t|t+1}$ and \widehat{gap}_t will co-vary positively over time. It is thus able to account for this qualitative regularity in time series data on returns in asset markets. However, it renders no predictions about whether the premium will rise or fall or what the exact size of the gap effect might be in the coming months.

The unfolding of $\widehat{pr}_{t|t+1}$ over time depends on how P_t moves relative to $\widehat{P}_t^{\text{BM}}$, which, as we show in the next section, depends on how individuals’ price forecasts in the aggregate unfold. The qualitative and contingent restrictions that we use to portray change in $\widehat{P}_{t|t+1}$, like the gap restrictions, allow for myriad possible revisions of individuals’ forecasting strategies. This approach of partially predetermining change leads to the key feature of the model:

³⁴See Frydman and Goldberg (2008, 2013) for a simple algebraic example of partially predetermined probability distributions.

- its predictions concerning whether P_t and $\hat{p}r_{t|t+1}$ will rise or fall in any one period are contingent on how forecasting strategies unfold.

As we discuss in section 6, this knowledge contingency enables the model to recognize the coexistence of both bulls and bears without presuming that either group is obviously irrational.

4.3 The Gap Model and Momentary Equilibrium

Endogenous prospect theory, portfolio balance, and the gap conditions in (20) imply the following equilibrium condition for price:

$$P_t = \hat{P}_t^{\text{BM}} + \frac{1}{(1 + \sigma_t)} \left(\hat{P}_{t|t+1} - \hat{P}_t^{\text{BM}} \right) \quad (23)$$

Like most other asset market models, this specification implies that the main driver of price fluctuations in asset markets is individuals' price forecasts. It shows that P_t would undergo a protracted swing away from benchmark levels during periods of time in which $\hat{P}_{t|t+1}$ moved persistently away from these levels.³⁵ This swing would end when the swing in $\hat{P}_{t|t+1}$ ended.

5 Bubbles and Lost Fundamentals: Artifacts of the Contemporary Approach

Standard REH models of asset markets suppose that individuals' price forecasts are based on fundamentals, such as such as corporate earnings and interest rates. The inability of these models to account for the wide price swings in asset markets, and the belief that they provide the right way to portray the importance of fundamentals for rational forecasting, has led economists to develop so-called "bubble" models. In these models, individuals' price forecasts are driven over the short-term not by market fundamentals, but by speculative manias, crowd psychology and other psychological biases, or technical momentum trading. Such forecasting behavior, which behavioral-finance portrays with mechanical rules, leads participants to bid an asset price increasingly away from levels that are consistent with fundamental considerations.³⁶

³⁵We are assuming that any changes in σ_t do not outweigh the impact of a change $\hat{P}_{t|t+1}$. This assumption is consistent with the qualitative and contingent conditions that we impose on how market participants revise their forecasting strategies. See section 6.2.

³⁶See Blanchard and Watson (1982) for an early REH bubble model. For seminal behavioral bubble models, see Frankel and Froot (1987) and De Long et al. (1990). More recent models include Abreu and Brunnermeier (2003) and De Grauwe and Grimaldi (2006).

Many point to the long upswing in US equity prices during the 1990's as a prime example of such behavior, and widely refer to this upswing as the "dot.com or internet bubble." During this period, there was indeed much confidence and optimism, and even a sense of euphoria, about Internet stocks, with initial public offerings for many companies witnessing remarkable price increases.³⁷ At its height in August 2000, the broader S&P 500 price index had climbed to roughly 43 times its underlying earnings. This eclipsed the market's valuation in October 1929 of 33 times earnings, which had stood as the market's all-time high until the 1990's.

But, beyond their epistemological flaws,³⁸ many bubble models are simply inconsistent with the basic features of the price swings we actually observe in asset markets. According to most of these accounts, prices are supposed to rise steadily, save for occasional random movements in the opposite direction, and when the speculative fever dissipates and the bubble bursts, prices are supposed to jump immediately back to their "known true" fundamental values. However, the long swings shown in figures 1 and 2 all involve extended periods (sometimes lasting months) during which the asset price undergoes a persistent, but partial, movement back towards the benchmark. Moreover, the sustained reversals that eventually arise do not involve an immediate return to benchmark values; some of these reversals last for years at a time and eventually shoot through those benchmarks.³⁹

5.1 Technical Trading and Psychology's Inability to Sustain Swings

To be sure, many participants in markets make use of technical rules and this trading can contribute to price trends. But the bubble notion's assumption that such considerations alone could sustain an upswing last-

³⁷Globe.com and eToys, just to name two, saw price rises on the first day of trading of 606% and 280%, respectively. During October 1999, the six largest technology-related companies – Microsoft, Intel, IBM, Cisco, Lucent, and Dell – had a combined market value of \$1.65 trillion, or nearly 20% of US gross domestic product.

³⁸See our companion paper and Frydman and Goldberg (2013), and references therein.

³⁹Some behavioral bubble models imply long-lasting upswings and downswings, for example, Frankel and Froot (1987). An open question is whether these and learning models (e.g., Mark, 2007) can account for the persistence of asset price fluctuations relative to benchmark values. Johansen et al. (2010), Frydman et al. (2012) and others find that real exchange rates are near- $I(2)$. However, Johansen and Lange (2011) show that Blanchard and Watson's (1982) REH bubble model implies at most near- $I(1)$ persistence because of its bust dynamics. De Grauwe and Grimaldi (2006) simulate an extension of Frankel and Froot's model and find that it is consistent with currency fluctuations that are near- $I(1)$.

ing an entire decade is implausible. Technical trading mostly takes place over minutes or hours. Schulmeister (2003) points out that the technical rules that are used in markets differ in terms of how quickly they generate a buy or sell signal once a price trend has already started. This can lead speculators to prolong an initial price trend. However, the trigger times of most technical market strategies differ only in terms of hours or days. Such speculation simply cannot account for long swings lasting many months or years.

Psychological factors, such as confidence and optimism, also influence decision-making and no doubt played a role in leading participants in the aggregate to bid up stock prices in the 1990's. But, purely psychological accounts of asset markets overlook the possibility that to forecast price movements, participants look to fundamental factors that they think will move the market. Moreover, psychological factors themselves are influenced by fundamental considerations. It is simply implausible to suppose that pure crowd psychology could sustain long price swings lasting many years. Indeed, any confidence and optimism that might exist in the stock market would quickly evaporate if, say, earnings and overall economic activity consistently moved in the opposite direction.

5.1.1 Fundamental Underpinning of Psychological and other Non-Fundamental Considerations

Technical and psychological considerations are difficult to measure and incorporate in formal statistical analysis of the determinants of asset price fluctuations. However, using textual data that is compiled from *Bloomberg News's* daily "market-wrap" (end-of-day) stories, Mangee (2011) is able to measure the influence of these considerations, as well as the influence of a wide range of fundamental factors, on daily stock price fluctuations.⁴⁰ In writing market-wrap stories, Bloomberg's journalists rely on contacts with 100-200 fund managers and other actors directly involved in the markets. Every one of their wrap stories includes at least one direct quote from one or more of these individuals concerning their views about the key factors driving the market. These stories thus provide a virtual window into the decision-making of the professional players whose trading determines prices.

Bloomberg journalists indicate that psychological considerations, such as confidence, optimism, and fear, play an important role in daily price fluctuations, having mentioned them as a main driver of prices on 55% of the days on average over the sample.⁴¹ However, when Mangee (2011)

⁴⁰The study covers the period January 4, 1993 (Bloomberg's first report) through December 31, 2009.

⁴¹Two excerpts from Bloomberg's wrap stories illustrate how psychological factors

examines the influence of pure psychology – psychological factors that are mentioned separately from fundamentals – he finds that they were mentioned on only 1 % of the days on average.⁴²

Bloomberg’s wrap stories indicate that technical momentum trading also played a relatively minor role for stock price fluctuations. Mange (2011) finds that this type of trading was mentioned as a main driver of stock prices on only 2 % of the days on average over the sample.⁴³

Even when Mangee (2011) combines the influence of pure psychology and momentum trading, there is little support for the bubble view of swings. In virtually no cases do these bubble considerations alone move the market. Moreover, their role in accelerating swings seems marginal. This finding is illustrated in figure 4, which plots the average proportion of days per month that any one of these bubble considerations was mentioned.⁴⁴

The figure shows that bubble considerations had their greatest impact in the second half of the 1990’s, providing perhaps some evidence of the bubble view of long upswings. However, these considerations were hardly mentioned at all during much of the 1990’s upswing in stock prices. Their importance did rise sharply beginning in 1997. But a high mark of 9 % means that they were mentioned on less than two out of 20 trading days per month on average. Moreover, the importance of bubble considerations began falling rapidly in February 1999. This was an entire year prior to the sharp reversal in equity prices in mid-2000, during the most excessive part of the upswing and exactly when the

are reported. In one story (April 21, 2009), a money manager for Pilgrim Investments reports that “IBM earnings are extremely positive...this will give confidence and stability to the market.” In another story (March 2, 2009) titled “US Stocks Tumble Trimming Gains at End of Three Week Rally,” a money manager observes that “You have got a lot of fear going into earnings.”

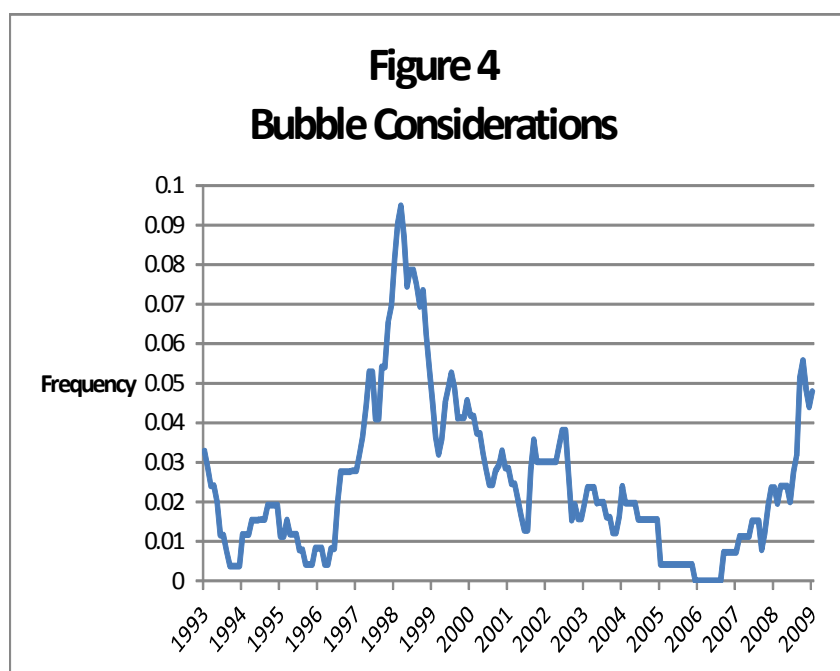
⁴²The reporting of pure psychological factors is illustrated by two excerpts from Bloomberg’s wrap stories. The market wrap on April 21, 1998 mentions the observations of a chief investment officer for BankBoston Corp, who remarked that “I do think it’s mania...anytime stocks appreciate 30 to 50 percent in a day, it’s the greater fool theory. People think there will always be someone who will pay a higher price.” In another story (August 4, 1998), the same investment officer observes that “The selling is feeding on itself...people are indifferent about stock prices and valuations. Now they’re fearful.”

⁴³The reporting of technical momentum trading is illustrated by two excerpts from Bloomberg’s wrap stories. The market wrap on January 11, 2001 reports that “The Nasdaq extended gains after 1 pm surging more than 2 percentage points in an hour, as ‘momentum’ investors, or those who make short term bets on a stock’s direction, rushed to buy shares, traders said.” In another story (October 4, 2001), a money manager observes that “So-called momentum investors have been buying technology shares because they have to get their foot back in the door and not get left behind.”

⁴⁴The time plot in figure 4 is based on a 12-month moving average.

bubble view would imply that pure psychology and momentum trading should have their greatest impact. Whatever their impact on prices, these considerations were not the main drivers of the upswing.

Even if one were to view this evidence that psychology and momentum considerations alone do not drive asset-price movements as “too soft” to constitute a formal rejection of the bubble view, there is an abundance of evidence showing the central role that fundamental factors play in asset markets.



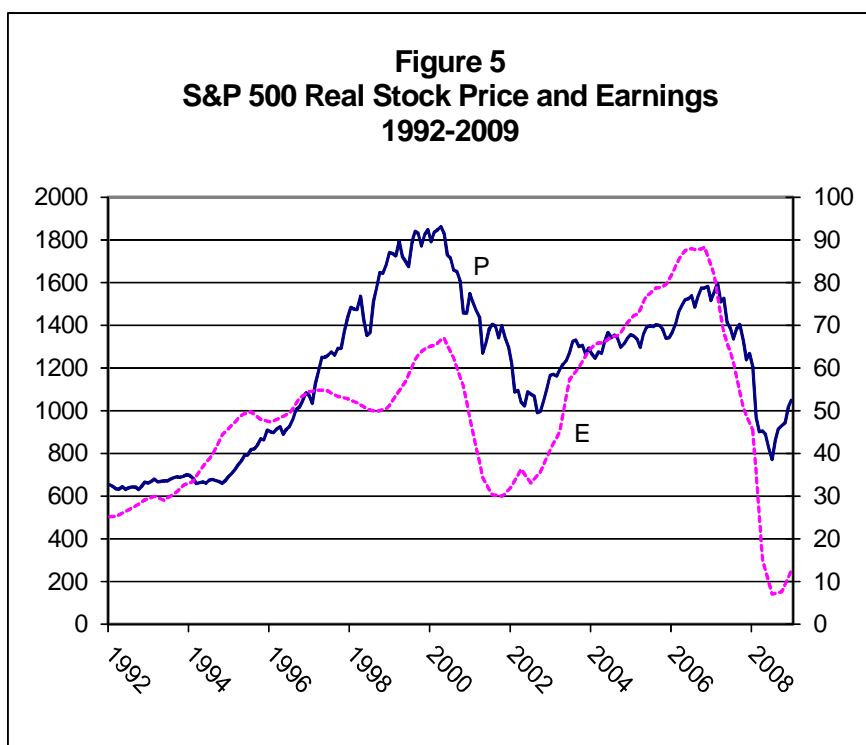
5.2 Fundamentals Matter, But in Non-Routine Ways

One only has to watch *Bloomberg Television* or *CNBC* for a week or two to realize that news on a wide range of fundamental factors drives prices in major asset markets. As earnings announcements are made or policy developments in Washington, D.C. become known, one sees the markets react. One also sees that the fundamentals that matter change over time in non-routine ways.

During much of the 1990’s, for example, corporate earnings, GDP, employment, exports, productivity levels, and other economic indicators were rising strongly, while inflation rates were declining or holding at benign levels. Free-trade agreements and other political and institutional changes, together with loose monetary policy, were also conducive to growth. As these developments unfolded, they no doubt reinforced

confidence and optimism in the widespread view that the US and other economies were in the midst of an information-technology revolution. The bullish trends in fundamental factors, and the greater confidence that they engendered, led many market participants to raise their price forecasts, thereby bidding up stock prices.

The importance of fundamental factors in driving asset prices is easily seen in figure 5, which plots the S&P 500 price index along with the basket's underlying earnings. The co-movement of the two series is striking. Not only do the broad swings in the series rise and fall together, but their major turning points in 2000, 2003, and 2007 are closely synchronized. The figure belies the bubble view that long upswings in stock prices away from benchmark levels are unrelated to fundamental factors.



Not surprisingly, *Bloomberg's* wrap stories point to the importance of fundamentals in sustaining swings.⁴⁵ Manglee's (2011) data show that at

⁴⁵We again illustrate Bloomberg's reporting with excerpts from two of its wrap stories. The story of April 18, 2001 reports that "US stocks rallied after the Federal Reserve surprised investors by cutting interest rates for the fourth time this year. In a story on March 1, 2004, the chairman of Walnut Asset Management LLC observes that "The environment is pretty doggone good for stocks...earnings appear to be stronger than anticipated."

least one fundamental factor was mentioned as a driver of stock prices on virtually every day in his sample. The top four broad categories of fundamental considerations were earnings, the economy, interest rates, and sales. A factor in one of these categories was mentioned on 65%, 47%, 38%, and 23% of the days on average over the sample, respectively.⁴⁶

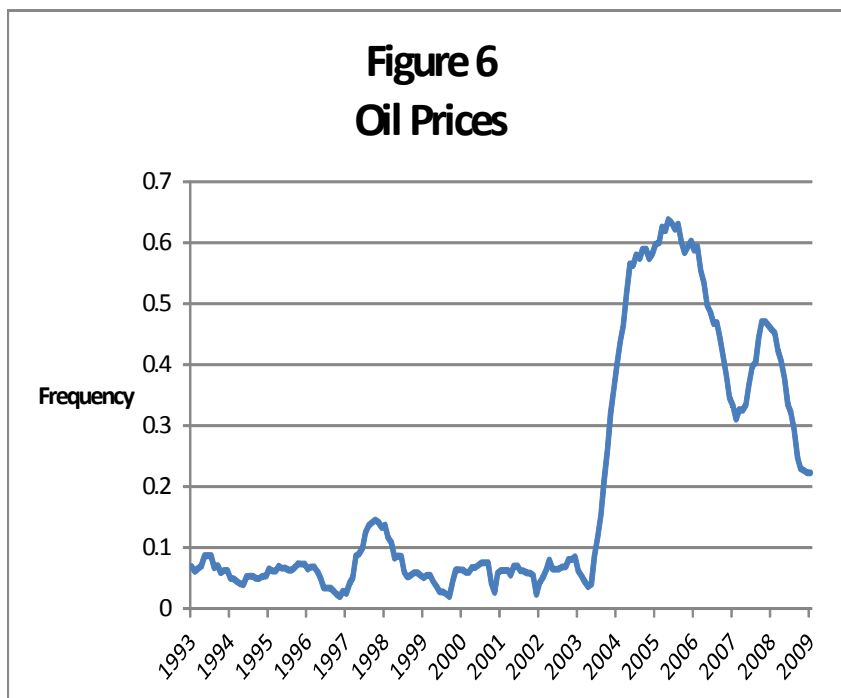
We have argued that the causal process underpinning asset prices changes in non-routine ways, in part because profit-seeking participants must rely on imperfect knowledge and psychological considerations in revising their forecasting strategies. *Bloomberg's* wrap stories show unambiguously this non-routine change. Indeed, they indicate that the changing nature of the fundamental relationships driving asset markets take a striking form: different variables matter for prices during different time periods. For example, figure 6 plots the average proportion of days per month that oil prices were mentioned as a main driver of stock prices.⁴⁷ The figure shows that the market did not pay much attention to the oil price until the end of 2003, when its importance began rising dramatically. By the end of 2004, 60% of each month's wraps mentioned this factor as a driver of the market. No one could have fully foreseen the timing and magnitude of this rise.

Empirical studies that allow for structural change without fully pre-specifying it find a similar result. They report that although the causal process driving asset markets changes in non-fully predetermined ways, there are extended time periods during which the non-routine change that does occur is sufficiently moderate that a relatively stable relationship between an asset price and a set of fundamental variables can be estimated. They find not only that fundamental factors matter over the short-term, but that different sets of fundamentals matter during different time periods.⁴⁸ Fully foreseeing when such distinct time periods might occur or how long they might last, let alone the precise nature of the fundamental relationships during those periods is simply beyond everyone's reach.

⁴⁶Mangee's measure of the frequency with which earnings considerations are mentioned by *Bloomberg's* wrap stories includes company announcements of earnings and earnings forecasts. It also includes mentions of stock-price movements that are reported to have arisen because other informational variables, for example interest rates or sales, led participants to revise their earnings predictions.

⁴⁷Again, a 12-month moving average is used.

⁴⁸For stock prices, see Mangee (2011), and for exchange rates, see Goldberg and Frydman (1996a,b) and Beckman et al. (2011).



6 An IKE Account of Asset Price Swings

The importance of fundamentals for short-term price movements in asset markets and the central role that individuals' price forecasts play in the process (portrayed by equation 23) indicate that prolonged price swings stem from trends in fundamentals and how individuals revise the ways they interpret these trends in thinking about the future. The microfoundations of our account of asset price swings recognize the importance of fundamentals for forecasting and, like before, involve qualitative and contingent restrictions to portray revisions of forecasting strategies.

Market participants' imperfect knowledge and the influence of psychological considerations on their decision-making implies that when and how they revise their forecasting strategies does not conform to any mechanical rules. Moreover, a key feature of price swings in asset markets is that they are irregular in duration and magnitude: some swings last for months while others last for years, some involve small departures from benchmark values, while such departures are large during others. No one can predict precisely when a price swing might begin or when it might end.

In building the microfoundations for a model of this aggregate regularity, we look for regularities on the individual level that are similarly contingent. In portraying how individuals revise their strategies

for forecasting price we formalize a qualitative regularity that often, but not always characterizes behavior. Consequently, our representation is not only qualitative, but contingent: although the qualitative conditions that we use to portray individual forecasting are assumed to hold most of the time, we do not prespecify the points when they do not. This contingency enables us to account for the unpredictable irregularity of asset price swings and the non-routine way that fundamentals underpin them. It also allows us to recognize that bulls and bears coexist in the market and that psychological considerations play a role in decision-making without presuming that individuals forego obvious profit opportunities.

6.1 An Individual's Price Forecast

We portray an individual's point forecast of next-period's stock price as

$$\hat{P}_{t|t+1}^i = \beta_t^i Z_t^i \quad (24)$$

where the vector Z_t^i represents the informational variables that individual i uses in forming her price forecast and β_t^i represents the vector of weights that she attaches to them in thinking about the future. This representation implies that there are two key factors that underpin the evolution of an individual's price forecast over time: revisions of her forecasting strategy – changes in β_t^i , which could include changes in the composition of the set of causal variables Z_t^i – and movements in the informational variables.

To illustrate our model of asset price swings, we begin with the assumption that the informational variables in the model follow random walks with constant drifts:⁴⁹

$$\Delta Z_t^i = \mu^i + \epsilon_t^i \quad (25)$$

where μ^i and ϵ_t^i are the vectors of drifts and white noise errors. There is much evidence that many of the fundamental variables that drive asset markets – for example, income levels, earnings, and inflation rates – are well approximated as unit-root processes.

The processes underpinning the unfolding of these fundamentals, of course, depend on economic policy and other features of the social context. Economic policy does change from time to time as new policy makers take charge or as policy makers' understanding of economic and social conditions evolves. These conditions themselves also change at times in non-routine ways. To represent such change in the model we

⁴⁹To simplify, we assume that the set of information variables does not include the asset price. Recognizing the importance of this variable does not alter the main conclusions of our analysis. See Frydman and Goldberg (2007).

could allow for time-varying drifts in the Z processes in (25). This would enable us to capture the kind of behavior we have already seen in corporate earnings (figure 5): this variable trends in one direction for extended periods of time, followed by periods of time in which it trends in the opposite direction.⁵⁰

As with market participants' forecasting, we would look for qualitative and contingent regularities in the social context that might help in accounting for the irregular price swings. However, to highlight how IKE representations of forecasting behavior help to account for this aggregate regularity, we maintain the assumption of constant drifts for most of this section. We refer to this assumption as a "fixed policy environment."⁵¹

6.2 Non-Routine Revisions of Forecasting Strategies

We recall from the equilibrium condition in (23) that a price swing away from or toward benchmark values arises in the model during any stretch of time in which individuals' price forecasts move persistently away from or toward these values. The swing ends when the swing in $\hat{P}_{t|t+1} - \hat{P}_t^{\text{BM}}$ ends. Estimates of common benchmark values, such as those discussed in section 2, move much more slowly than the asset price. Thus, in order to focus on the central role of price forecasts in driving swings in asset markets, we set the aggregate of individuals' estimates of the benchmark equal to the constant, \hat{P}^{BM} .⁵²

In modeling revisions of forecasting strategies, we again rely on Keynes's (1936) account of asset markets. This account is often invoked by those who support the view that underlies bubble models, namely that asset prices are driven by purely psychological considerations. Indeed, Keynes repeatedly alludes to psychological considerations in describing speculation and investment, referring, for example, to "confidence with which we... forecast" (Keynes's, 1936, p. 148), "mass psychology" (Keynes's, 1936, p. 154), "spontaneous optimism" (Keynes's, 1936, p. 161), and "animal spirits" (Keynes's, 1936, p. 162).

However, we argue in Frydman and Goldberg (2011) that although

⁵⁰In fact, there is much evidence that the processes underpinning many macroeconomic time series are better characterized as involving a persistent drift rather than a constant or no drift, thereby implying near- $I(2)$ behavior. For evidence of such behavior in macroeconomic time series, see Johansen (1997), Kongsted and Nielsen (2004), Juselius (2007), Johansen et al. (2010), and Frydman et al. (2012).

⁵¹In section 6.5.1, we show how shifts in the policy environment help to explain the boundedness of asset price swings.

⁵²Movements of the imperfectly known benchmark are likely to play a significant role in some markets. However, in this chapter we focus on the fluctuations around benchmark levels.

psychological factors undoubtedly played a role in Keynes's thinking about markets, there is much in his *General Theory* to suggest that fundamental considerations were also important to his view of asset price fluctuations. Indeed, he begins his analysis of asset markets by discussing how "expectations of prospective yields" are, in the first place, rooted in individuals' understanding of fundamentals or

"knowledge of the facts which will influence the yield of the investment...[and the] existing market valuation... will only change in proportion to changes in this knowledge." (Keynes, 1936, p. 152)

In using their "knowledge of the facts" to form forecasts, participants

"fall back on what is, in truth, a convention... [which] lies in assuming that the existing state of affairs will continue indefinitely, except in so far as we have specific reasons to expect a change." (Keynes, 1936, p. 152)⁵³

Keynes's insight that market participants tend to assume that the "existing state of affairs will continue" suggests that they tend to stick with a forecasting strategy for stretches of time. Indeed, it is often unclear whether one should alter her strategy. A quarter or two of poor forecasting performance may be the result of random events rather than an indication of a failing strategy. So, unless an individual has "specific reasons to expect a change" in the market, she may leave her current strategy unaltered – even if its performance begins to flag over several periods. Moreover, even armed with "specific reasons to expect a change," it is entirely unclear what new forecasting strategy, if any, she should adopt.

We thus represent how participants tend to alter their thinking about how fundamentals matter with an empirical regularity that we refer to as "guardedly moderate revisions": there are stretches of time during which participants either maintain their strategies or revise them gradually. Such revisions do not generally alter, in substantial ways, the set of fundamentals that participants consider relevant or their interpretation of these fundamentals' influence on future outcomes. As we shall see, all that is needed to sustain a price swing during these time periods is for fundamentals to trend in unchanging ways, which they do quite often.

But, like price swings themselves, the tendency toward guardedly moderate revisions is a qualitative regularity that occurs in contingent

⁵³By "existing state of affairs," Keynes means "knowledge of the facts."

ways. There are occasions when news about fundamentals and price movements leads participants to revise their forecasting strategies in non-moderate ways. Such revisions can have a dramatic impact on price and spell the end of a price swing in one direction and the start of a new one in the opposite direction.

6.2.1 Formalization of Guardedly-Moderate Revisions

In formalizing the contingent regularity of guardedly-moderate revisions, we need to define a baseline against which revisions may be judged to be either moderate or non-moderate. We note that as trends in fundamentals unfold, an individual would, in general, alter her price forecast even if she were to keep her forecasting strategy completely unchanged. This status quo change in $\widehat{P}_{t|t+1}^i$ serves as our baseline for judging whether revisions are guardedly moderate.

Given the representations in (24) and (25), the total change in an individual's price forecast can be expressed as follows:

$$\widehat{P}_{t|t+1}^i - \widehat{P}_{t-1|t}^i = \mathcal{T}\widehat{P}_{t|t+1}^i + \epsilon_t^i \quad (26)$$

where $\mathcal{T}\widehat{P}_{t|t+1}^i$ is the “trend change” in the individual's forecast between $t - 1$ and t ,

$$\mathcal{T}\widehat{P}_{t|t+1}^i = \Delta\beta_t^i Z_t^i + \beta_{t-1}^i \mu^{Z^i} \quad (27)$$

This trend change depends on how an individual revises her forecasting strategy, $\Delta\beta_t^i$, and on what we call the “baseline drift” in her point forecast, $\beta_{t-1}^i \mu^{Z^i}$.⁵⁴ The baseline drift represents the status quo change in our model, which would result if an individual were to use the same forecasting strategy at $t - 1$ and t and the only change in her price forecast at t resulted from drifts in fundamentals. If she were to refrain from revising her strategy at the next T points in time, $\mathcal{T}\widehat{P}_{t|t+1}^i$ would again be equal to $\beta_{t-1}^i \mu^{Z^i}$ between each of these points. During this stretch of time, $\widehat{P}_{t|t+1}^i$ would tend to move in one direction, which would be determined by the algebraic sign of the baseline drift.

Suppose, for example, that an individual's $\beta_{t-1}^i \mu^{Z^i}$ was positive. In this case, movements in fundamentals would, on average, lead her to raise her price forecast between $t - 1$ and $t + T$.

Of course, movements in fundamentals may lead an individual to revise her forecasting strategy at any point during this stretch of time.

⁵⁴To portray change in the set of fundamentals that an individual might use in forming her price forecast, we suppose that the vector Z_t^i represents at every point in time all possible fundamentals that she might use. If, for example, an individual uses a particular variable Z_j^i to form her forecast at t but not at $t - 1$, then $\beta_{j,t-1}^i = 0$ and $\Delta\beta_{j,t}^i = \beta_{j,t}^i$.

To see how this works in our model, consider such a revision at t , which is portrayed by a new set of parameters, $\beta_t^i \neq \beta_{t-1}^i$. The impact of the new parameters on the price forecast at t is $\Delta\beta_t^i Z_t^i$. But, the revision also gives rise to a new baseline drift, $\beta_t^i \mu^{Z^i}$, which portrays how trends in fundamentals influence the individual's forecast in the next period, $t + 1$.

Each of these effects on the price forecast – $\Delta\beta_t^i Z_t^i$ at t and $\beta_t^i \mu^{Z^i}$ at $t + 1$ – could reinforce or impede the influence of the initial baseline drift, $\beta_{t-1}^i \mu^{Z^i}$, which in our example is positive. The new β_t^i would be reinforcing at t and $t + 1$ if $\Delta\beta_t^i Z_t^i > 0$ and $\beta_t^i \mu^{Z^i} > \beta_{t-1}^i \mu^{Z^i}$. In this case, the revision at t would add to the initial influence of fundamentals and contribute to a higher $\widehat{P}_{t|t+1}^i$ at t and a faster growing $\widehat{P}_{t|t+1}^i$ at $t + 1$.

If, however, the new β_t^i was impeding at t and $t + 1$ – $\Delta\beta_t^i Z_t^i < 0$ and $\beta_t^i \mu^{Z^i} < \beta_{t-1}^i \mu^{Z^i}$ – it would subtract from the positive influence of fundamentals at t and lead to a lower baseline drift in $t + 1$. The size of these effects could be large enough that, although the initial influence of fundamentals was positive, $\widehat{P}_{t|t+1}^i$ fell at t and $t + 1$.

But, regardless of whether revisions were reinforcing or impeding between $t - 1$ and $t + T$, if their effects on an individual's price forecast and baseline drift were moderate enough throughout this period, the trends in fundamentals would still tend to dominate and lead her to raise her price forecast in one direction.

This reasoning underpins our formalization of guardedly moderate revisions, which involves two qualitative conditions,

$$|\Delta\beta_t^i Z_t^i| < \delta_t^i \quad (28)$$

$$|\Delta\beta_t^i \mu^{Z^i}| < \delta_t \quad (29)$$

where $|\cdot|$ denotes an absolute value and $\delta_t^i = |\beta_{t-1}^i \mu^{Z^i}|$ is the magnitude of the baseline drift. The first condition constrains revisions at t so that the impact of movements in fundamentals on an individual's price forecast tends to outweigh the impact from revisions. The second condition also constrains revisions at t and implies that the sign of the baseline drift does not change. Consequently, if these conditions were to characterize how an individual revised her forecasting strategy over a stretch of time, and trends in fundamentals remained unchanged over that time, she would tend to alter her price forecast in one direction or the other.

The qualitative constraints in (28) and (29) embody the idea that unless an individual has “specific reasons to expect a change” in the “existing state of affairs,” she tends to maintain her forecasting strategy or, if she revises it, to do so in ways that would not have a large

impact on her price forecast. Because they restrict neither the causal variables that may enter the representation in (24), nor how exactly these variables might matter, these constraints leave room for non-rule-based revisions. Moreover, they do not imply that the change in an individual’s price forecast must be small in size: our model’s constraints are compatible with large changes $\widehat{P}_{t|t+1}^i$ that result from large movements in fundamentals.

6.2.2 Psychological Motivation for Guardedly-Moderate Revisions?

Although we find compelling Keynes’s insight that individuals fall back on the guarded assumption that the “existing state of affairs will continue,” we are unaware of any direct empirical evidence that would support this claim. To be sure, psychologists have uncovered experimental evidence of a regularity they call “conservatism”: when individuals change their forecasts about uncertain outcomes, they tend to do so gradually, relative to some baseline – a finding that is consistent with our characterization of guardedly moderate revisions.⁵⁵ But it is a finding that pertains to how an individual alters her *forecast* rather than to how she might revise her forecasting *strategy*.

Even if we were to formulate conservatism as a qualitative regularity, it would imply a restriction on an individual’s forecasting behavior that was stronger than our guardedly-moderate restrictions. For example, suppose that an individual kept her forecasting strategy unchanged between two consecutive points in time, but increased her price forecast nonetheless because of movements in fundamentals. Conservatism would restrict the size of the impact on $\widehat{P}_{t|t+1}^i$, whereas our guardedly moderate conditions would not.

Moreover, because behavioral economists insist on sharp predictions, they characterize conservatism not as a qualitative regularity but as a mechanical one. As a result, existing formulations of conservatism imply restrictions on individual behavior that are much stronger than our representation of guardedly moderate revisions. These fully predetermined representations presume that individuals under-react to new information in a fixed way relative to what an economist’s overarching probability model would imply. They thus imply that individuals never revise their forecasting strategy.⁵⁶

⁵⁵See Edwards (1968) and Shleifer (2000) and references therein.

⁵⁶See, for example, Barberis et al. (1998).

6.3 Irregular Swings in an Individual's Price Forecast

Market participants's tendency to deal with uncertainty by keeping their forecasting strategies unaltered or revising them in guardedly moderate ways is a *contingent* regularity: it characterizes behavior during stretches of time that begin and end at moments that no one can predict. Eventually the "existing state of affairs" changes or an individual has "specific reasons to expect" such a change. Even if trends in fundamentals continued in the same broad directions, the qualitative regularity of guardedly moderate revisions may cease to hold at unpredictable moments of time.

Consequently, our IKE representation does not restrict an individual's forecasting behavior to be consistent with the conditions in (28) and (29) at every point in time. Moreover, it does not prespecify the stretches of time during which they characterize behavior, that is, when these stretches might begin or end. Because of this contingency, our model implies that an individual's price forecast will undergo swings of irregular duration and magnitude.

To see this, consider a stretch of time from $t-1$ to $t+T$ during which trends in fundamentals remained unchanged and an individual's initial interpretations of these trends would, in the absence of any revisions of her strategy, lead her to raise her price forecast. If, during this stretch, revisions of her strategy could be characterized as guardedly moderate, her price forecast would tend to rise over the period. An end to this upswing in $\hat{P}_{t|t+1}^i$ would occur at $t+T+1$, if her revisions were impeding and non-moderate – that is, *not* characterized by the conditions in (28) and (29). Such drastic revisions would have a greater impact on her price forecast than the impact from the drifts in fundamentals, and so $\hat{P}_{t|t+1}^i$ would tend to fall at $t+T+1$. Moreover, non-moderate and impeding revisions would lead to a switch in the sign of the baseline drift, so the influence of these drifts on our individual's price forecast in the next period, $\beta_{t+T+1}^i \mu^{Z^i}$, would be in the down direction. The fall in $\hat{P}_{t|t+1}^i$ would then tend to continue if revisions of the individual's strategy were once again characterized as guardedly moderate for another stretch of time beyond $t+T+1$.

The downswing in $\hat{P}_{t|t+1}^i$ would continue until the individual once again decided to revise her strategy in a dramatic and impeding way. At such points in time, her forecast would experience another reversal, which may or may not turn into a sustained counter-movement.

6.4 Bulls, Bears, and Irregular Swings in the Aggregate Forecast

There are, of course, stark differences in bulls' and bears' strategies; after all, one group forecasts a rise in price, while the other forecasts a decline. However, because our characterization of guardedly moderate revisions is qualitative and contingent, it is open to non-routine changes in bulls' and bears' forecasting strategies and consistent with the myriad diverse ways in which this change occurs. Regardless of the precise way in which a participant revised her strategy, or whether she was a bull or a bear, her price forecast would tend to move in one direction or the other for as long as her revisions continued to be guardedly moderate and trends in fundamentals remained unchanged.

The equilibrium condition in (23) shows that what matters for asset prices is how the aggregate of participants' diverse price forecasts moves over time. Even if all participants were assumed to revise their strategies in guardedly moderate ways over a stretch of time, this would not, in general, imply that the aggregate forecasting strategy was also characterized by guardedly moderate revisions.

In order to connect the implications of our representation of behavior on the individual to aggregate outcomes, we make use of assumptions concerning the degree to which *qualitative* interpretations about movements in fundamentals varies across individuals. This variation implies another layer of contingency to the predictions of our IKE model.

6.4.1 Moving to the Aggregate Level

To see what assumptions about diversity are needed for our model to generate predictions on the aggregate level, we initially consider a stretch of time, t to $t + T$, during which the drifts in fundamentals remain unchanged and all participants revise their forecasting strategies in guardedly moderate ways. Participants who are bulls at t predict a higher price at $t + 1$,

$$\widehat{P}_{t|t+1}^{i,L} - P_t > 0 \tag{30}$$

while those who are bears predict a lower price,

$$\widehat{P}_{t|t+1}^{i,S} - P_t < 0 \tag{31}$$

But, regardless of whether they are bulls or bears at t , we know from section 6.2.1 that all the individual $\widehat{P}_{t|t+1}^i$'s in our example move in one direction or the other, on average, between t and $t + T$.

Whether an individual's price forecast rises or falls over the period depends on how the trends in fundamentals initially impact her forecast, which we portray by the initial baseline drift. If, for example, this initial

impact is positive ($\beta_{t-1}^i \mu^{Z^i} > 0$), all subsequent trend changes in an individual's forecast will be positive ($\mathcal{T}\widehat{P}_{t|t+1}^i > 0$) and she will tend to raise $\widehat{P}_{t|t+1}^i$ over the period. We note that such an upswing in $\widehat{P}_{t|t+1}^i$ could characterize an individual who is a bear throughout the period. Although a bear's interpretation of fundamentals implies $\widehat{P}_{t|t+1}^i < P_t$, movements in fundamentals – say, a rise in overall economic activity – could well lead her to increase $\widehat{P}_{t|t+1}^i$, thereby becoming less bearish over time.

In the aggregate, the trend change in $\widehat{P}_{t|t+1}$ at each point in time is a weighted average of the individual $\mathcal{T}\widehat{P}_{t|t+1}^i$'s given in equation (27):

$$\mathcal{T}\widehat{P}_{t|t+1} = \sum_i \omega^i \left(\Delta\beta_t^i Z_t^i + \beta_{t-1}^i \mu^{Z^i} \right) \quad (32)$$

where the aggregation weights, as before, are based on wealth shares and assumed to be constant. It is immediately clear that if trends in fundamentals and guardedly-moderate revisions led all individuals to alter their price forecasts in the same direction between t and $t+T$, the aggregate $\widehat{P}_{t|t+1}$ would also undergo a swing.

To be sure, knowledge is imperfect and some market participants will interpret movements in certain fundamentals positively, whereas others view them negatively. Whether $\widehat{P}_{t|t+1}$ tends to move in one direction between t and $t+T$ in our example would then depend on the degree to which participants interpret movements of fundamentals in a qualitatively similar way. This diversity of qualitative interpretations varies over time in ways that no one can fully foresee. However, if it remained sufficiently small, then the constant trends in fundamentals and guardedly-moderate revisions on the individual level would lead most participants to alter their price forecasts in one direction over the period. Moreover, these swings in individual $\widehat{P}_{t|t+1}^i$'s would be associated with a swing in $\widehat{P}_{t|t+1}$ in the same direction.

In what follows, we impose this constraint on the variation in diversity as long as the asset prices' departures from estimates of the benchmark value are below a threshold level. Indeed, in most markets, participants tend to interpret many of the fundamentals that drive forecasts in a qualitatively similar way. For example, in the stock market, positive trends in overall economic activity, earnings, sales, and employment are widely viewed by participants as reasons to raise their price forecasts regardless of whether they are bulls or bears, whereas rising interest rates and oil prices have the opposite effect.

However, we would expect diversity in the market to grow if stock prices moved, for example, far above estimates of benchmark values.

Participants know that such departures are eventually reversed, although no one knows when the reversal will begin. Even when valuations become high, some participants continue to forecast that the upswing will persist over the next period, while others predict a reversal. Rising earnings and economic activity may lead both bulls and bears to raise their price forecasts, but at some point, even these trends would not be interpreted in qualitatively similar ways as bears begin to gain greater confidence in a reversal and stop raising their \hat{P}_{t+1}^i 's. As we discuss in more detail in the next section, this growth in diversity can spell the end of a price swing.

The assumption that all market participants revise their strategies in guardedly moderate ways is strong. At each point in time we would expect that the forecasting behavior of some participants could be characterize as guardedly moderate, while others could not. Whether the aggregate forecast tends to move in one direction between t and $t + T$ in our example depends on the relative weight of the participants in the market whose revisions are guardedly moderate. A swing would arise during the period if this weight remained sufficiently high.

6.5 Irregular Price Swings

As long as departures of the asset price from estimates of benchmark values remain below some unknown threshold, the model does not prespecify the stretches of time during which the trends in fundamentals are persistent and market participants, on the whole, revise their forecasting strategies in guardedly-moderate ways. It does not prespecify, therefore, when swings in the aggregate price forecast, \hat{P}_{t+1} , and thus in the asset price, will begin or end.

Participants decisions to revise their strategies depend on many considerations, including their current strategy's performance, whether they have "specific reasons to expect a change" in how fundamental factors are trending or how they are influencing prices, and the "confidence with which we... forecast" (Keynes, 1936, p. 148). Non-moderate and impeding revisions of forecasting strategies are often proximate to points at which trends in fundamentals also reverse or there is a major change in economic policies or institutions. At such points, the market would experience a price reversal. But, once such changes have occurred and been interpreted, individuals are likely to resume revising their strategies in guardedly moderate ways: they would tend to keep their forecasting strategies unchanged or revise them only moderately, but would always be on guard for change. In that case, and if fundamentals trended in persistent directions, the asset price would begin trending in one direction for another stretch of time.

The new price swing would continue until participants again lost confidence that the new trends in fundamentals would persist, or some other change in the existing state of affairs led them to revise their strategies in non-moderate and impeding ways. At such points in time, the asset price would experience another reversal, which may or may not turn into a sustained counter-movement.

6.5.1 Risk and Bounded Instability

Although our model does not fully prespecify when asset price swings begin or end, it does imply that swings away from estimates of benchmark values eventually end and are followed by sustained countermovements back toward these values. This implication follows from our account of risk and assumptions about the unfolding of diversity.

As an asset price rises well above or falls well below most participants' perceptions of benchmark values, those who are betting on further movements away from these levels raise their assessments of the riskiness of doing so. Eventually, these assessments lead them to revise their forecasting strategies in non-moderate and non-reinforcing ways. When that happens, even the most excessive price swings come to an end, and are followed by sustained reversals back toward benchmark levels.

Consider a stretch of time during which market participants revise their forecasting strategies in guardedly-moderate ways. A bull forecasts that the price swing will continue over the near-term, while a bear forecasts the opposite. However, our assumption about diversity implies that as long as the asset price does not depart too far from benchmark values, most bulls and bears would interpret movements in fundamentals in qualitatively similar ways. Suppose, initially, that the mix of forecasting strategies and drifts in fundamentals lead both bulls and bears to raise their price forecasts over time, that is, the baseline drifts for both groups, $\beta_{\tau-1}^{i,L} \mu^{Z^{i,L}}$ and $\beta_{\tau-1}^{i,S} \mu^{Z^{i,S}}$, are positive. These developments would lead bulls to become more bullish and bears to become less bearish. During this stretch of time, then, $\hat{P}_{t|t+1}$ and P_t , would tend to rise, say, farther above most estimates of benchmark levels.

But, although bulls expect a greater return, they understand that such upswings eventually end, so they increase their assessment of the risk of a reversal and capital losses. The resulting rise in their premiums tempers their desire to increase their speculative positions. If trends in fundamentals continued, thereby prolonging the excessive price swing, a threshold would eventually be reached at which bulls would become so concerned about a reversal that they would no longer revise their forecasting strategies in guardedly moderate ways. At that point, they would either reduce their long positions or abandon them altogether,

precipitating a price reversal.

Bears too understand that upswings eventually end, which is why they also change their premiums, but in the opposite direction. If the upswing continued, they would eventually cease to interpret trends in fundamentals as reasons to raise their price forecasts. But this would imply that they revised their forecasting strategies in non-moderate and impeding ways, likewise contributing to the self-limiting nature of long swings away from benchmark levels.

When, precisely, the gap from benchmark levels is perceived to be too large for bulls and bears to continue to revise their strategies in guardedly-moderate ways depends on many factors, including economic, political, and policy considerations that no one can fully prespecify. Thus, no one can fully prespecify when long swings away from benchmark levels will eventually end.

In currency markets, policy makers make use benchmarks such as PPP in setting economic policy and their actions play an important role in keeping exchange rate swings bounded. The empirical record shows that policy officials eventually become concerned about large departures in the exchange rate from PPP and alter policy to engender a reversal.⁵⁷ As with revisions of forecasting strategies, we would not expect that such policy changes would follow any mechanical rule. But, the fact that many of the major reversals in currency markets are proximate to major changes in policy suggests that policy makers play an important role in keeping long swings in currency markets bounded.⁵⁸

⁵⁷Examples of such behavior include the coordinated interventions by central banks and the changes in monetary and fiscal policies that were aimed at bringing down U.S. dollar exchange rates in 1985 and yen exchange rates in 1995, as well as the interventions by the U.S. Federal Reserve and the European Central Bank to stem the dollar's fall in 2007 and the first half of 2008.

⁵⁸For example, the major reversals in U.S. dollar exchange rates in late 1979 and early 1985 were associated with the arrival of Paul Volcker and James Baker, respectively, both of whom quickly engineered major changes in policy. An example of a connection between policy and major reversals in other asset markets is provided by the downturn in U.S. equity markets that began in August 2000, which came on the heels of the Federal Reserve's decision to raise the federal funds rate from 4.74% in July 1999 to 6.5% in May 2000. Beyond the policy channel, departures in asset prices from benchmark values influence trends in macroeconomic fundamentals endogenously in ways that also keep asset price swings bounded. For example, swings in exchange rates eventually lead to changes in current account imbalances and economic growth that would tend to limit such swings.

7 Contingent Predictions of Long Swings and Their Compatibility With Rationality

Despite its contingent character and openness to non-routine change, our representation of bulls' and bears' decision-making and assumptions about diversity place sufficient structure on the analysis to account for the basic features of asset price swings that we depict in figures 1 and 2.

The qualitative conditions formalizing guardedly-moderate revisions and our assumptions about diversity do not produce sharp predictions about asset-price fluctuations. Instead, they are consistent with myriad possible changes in β_t and the composition of Z_t between two consecutive points in time. At any point in time, therefore, they imply myriad post-change probability distributions for price – one for each possible value of β_t – conditional on any one of the distributions at time $t - 1$ and the processes underpinning fundamentals. However, each one of these partially predetermined distributions implies that price would tend to move in one direction or the other over time if the drifts underpinning fundamentals remained unchanged.

Our IKE model predicts, therefore, that price swings in asset markets will occur during stretches of time in which trends in market fundamentals are persistent, and participants, on the whole, interpret the impact of these trends on their price forecast in a qualitatively similar manner and revise their strategies in guardedly moderate ways. The uncertainty inherent in forecasting asset prices, as well as findings in psychology suggest that participants often stick with a strategy for stretches of time and that when they do revise their strategies, they are reluctant to do so in dramatic ways. Moreover, macroeconomic fundamentals often trend in particular directions for years at a time. Given these qualitative regularities in individual forecasting and the social context, our IKE model predicts that asset prices will undergo swings either away from or toward estimates of benchmark values quite often. If a price swing is headed toward benchmark values, and the qualitative conditions for a swing endure, which they often do, the asset price will eventually shoot through the benchmark and begin trending away from the other side. Moreover, participants' use of the gap from benchmark levels to assess the riskiness of their speculative positions implies that long swings away from these levels cannot last forever. Eventually conditions in the market change and a sustained reversal results.

The contingent nature of its representation on the individual level implies that the model renders no prediction of whether the asset price will rise or fall in any period. The trends in fundamentals could reverse directions at any point in time and participants may cease to interpret

them in a qualitatively similar manner or decide to revise their strategies in non-guardedly moderate ways at any point in time. Consequently, at each point in time, the model is consistent with many partially predetermined probability distributions that imply a tendency for the asset price to move in the same direction across consecutive points and with many others that imply a tendency for it to move in the opposite direction across consecutive points.

The model is thus compatible with the coexistence of both bulls and bears. At each point in time, it is reasonable for some participants to expect the price to rise, and for others to expect that it will fall. It may even be reasonable for some individuals to remain consistently bullish or bearish during a period of time in which the asset price moves steadily against them. Indeed, an individual might reasonably decide to increase the size of her long or short position precisely *because* the price has moved further away from her expected level.

8 An Intermediate View of Markets and the Role of the State

Our IKE model has enabled us to uncover the importance of both fundamentals and psychological considerations for understanding price swings and risk in financial markets without presuming that market participants forego obvious profit opportunities. In Frydman and Goldberg (2009, 2011), we show how this model leads to an intermediate view of asset price swings away from and back toward benchmark values: they play an integral role in the process by which financial markets evaluate prior investments and foster new companies and projects – the key to modern economies’ dynamism. And yet, owing to the imperfection of knowledge, these swings can sometimes become excessive, implying huge economic and social costs.

This intermediate view contrasts sharply with the polarized positions that are implied by fully predetermined models of macroeconomics and finance: asset price swings are based either on “rational” decision-making that enables society to allocate its scarce capital nearly perfectly or on crowd psychology and technical trading that allocates capital haphazardly.

Our intermediate view of markets not only sheds new light on the supposed empirical puzzles implied by fully predetermined models, but it also leads to a new way of thinking about the relationship between the market and the state. Because asset price swings away from benchmark values can sometimes become excessive, there is a role for the state to stand guard to dampen this excess. Our IKE model implies new channels for policy officials to accomplish this task and new tools with

which regulators can assess systemic and other financial-sector risks.

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