Imperfect Knowledge, Unpredictability and the Failures of Modern Macroeconomics

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Abstract

After re-iterating five well-known theorems about the properties of conditional expectations in stationary settings—such as providing unbiased minimum mean square error predictions despite incomplete information, and the law of iterated expectations—we clarify unpredictability and illustrate its prevalence empirically. We then relate unpredictability to imperfect knowledge about location shifts, and derive its implications. These include refuting the relevance of the five theorems about conditional expectations for the real world, entailing the failure of rational expectations and the invalidity of the inter-temporal mathematics underpinning dynamic stochastic general equilibrium models (DSGEs). Finally, we describe how to empirically model ever-changing worlds despite imperfect knowledge thereof.

JEL classifications: C51, C22.
KEYWORDS: Imperfect Knowledge; Unpredictability; Conditional Expectations; Location Shifts; Indicator Saturation; *Autometrics*.

1 Introduction

You are certain you are reading this paper (pace Descartes). But you are uncertain if this paper will be clear or informative. You may be uncertain as to the truth of some of its statements even after reading it. Uncertainty abounds, both in the world and in our knowledge about that world. Increased knowledge may help reduce our uncertainty, but imperfect knowledge is ubiquitous, both because the real world is immensely complicated and ever changing, and because many individuals seem to act as if 'ignorance is bliss' (see Lusardi, 2016) or worse still, to quote Isaac Asimov 'my ignorance is just as good as your knowledge' (column in *Newsweek*, 21 January 1980). Yet much of macroeconomics, and many empirical studies thereof, treat economies as if agents had complete knowledge on the basis of which they made informed judgements, and the resulting system was stationary and ever converging to an equilibrium. Critiques of such a view include Frydman and Goldberg (2007), Castle, Doornik, Hendry, and Nymoen (2014), Hendry and Muellbauer (2017) and Hendry (2017a) *inter alia*.

The intermittent failure of economic forecasts to 'foresee' the future reflects both imperfect knowledge and a non-stationary and evolving world that is far from 'general equilibrium' and closer to 'general

disequilibrium'. The theory of economic forecasting in Clements and Hendry (1999) analyzes such a world, and because the resulting processes are complicated, they also allow for the forecasting model being used to differ from the data generation process (DGP). This framework correctly reflects imperfect knowledge about many aspects of economies, especially when allowing for unanticipated sudden changes. They demonstrate that most of the well-known theorems about forecasting then fail, including showing that a forecast based on conditional expectations from a correctly-specified causal model need not be an unbiased minimum mean square error (MMSE) predictor: rather the converses of most of the theorems actually hold, including that *non-causal* forecasting devices can out-perform an in-sample causal model of the DGP.

Hendry and Mizon (2014) generalized these implications to reveal fundamental problems in the very mathematics used in inter-temporal economics, with disastrous consequences for dynamic stochastic general equilibrium (DSGE) systems, which transpire to be the least structural of all possible model forms as their derivation entails they are bound to 'break down' when the underlying distributions of economic variables shift.[1] This serious problem is highlighted by Hendry and Muellbauer (2017) in their critique of the **B**ank of **E**ngland **q**uarterly **e**conometric **m**odel (BEQEM–pronounced Beckem: see Harrison, Nikolov, Quinn, Ramsay, Scott, and Thomas, 2005), a DSGE which, as in the film 'Bend it Like Beckham', bent in the Financial Crisis, but so much that it broke and had to be replaced. Surprisingly, despite that abject failure, it was replaced by yet another DSGE (COMPASS: Central Organising Model for Projection Analysis and Scenario Simulation—see Burgess, Fernandez-Corugedo, Groth, Harrison, Monti, Theodoridis, and Waldron, 2013). Unfortunately, Fawcett, Masolo, Koerber, and Waldron (2015) showed that COMPASS had already failed to characterize data available before it was even developed. Persisting with such an approach introduces a triple whammy as:

a] the derivations sustaining DSGEs use an invalid mathematical basis;

b] imposing a so-called 'equilibrium' fails to take account of past shifts;

c] the DSGE approach assumes agents act in the same incorrect way as the modeller, so assumes agents have failed to learn that imperfect knowledge about location shifts forces revisions to their plans.

The structure of the paper is as follows. Section 2 reconsiders some famous theorems about conditional expectations, that are readily proved in stationary settings but are misleading in the context of real world economies. Section 3 discusses the concepts of uncertainty and unpredictability, and their links to imperfect knowledge. Section 4 illustrates empirical location shifts that have occurred and explains their prevalence. Section 5 uses imperfect knowledge about shifts to deconstruct the theorems about conditional expectations, and reveal the invalidity of their proofs facing location shifts. Section 6 briefly describes new modelling tools which are also designed to detect shifts, and Section 7 concludes.

2 Famous, but potentially misleading, theorems about conditional expectations

[1] The conditional expectation is the minimum mean square error (MMSE) unbiased predictor.

[2] The expectation of the conditional expectation is the unconditional expectation, which is the law of iterated expectations.

These are both well known, but [2] is re-proved in (9) below: see e.g., Goldberger (1991, p. 46–51).

[3] Incomplete knowledge of the conditioning information does not lead to biased expectations (see e.g., Clements and Hendry, 2005, and (4) for a proof).

[4] Conditional expectations can provide unbiased forecasts even in mis-specified, mis-estimated models (as shown e.g., by Hendry and Trivedi, 1972).

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[1]See http://voxeu.org/article/why-standard-macro-models-fail-crises for a less technical explanation.

[5] Replacing unknown expectations by realized future outcomes, as in New-Keynesian Phillips curve (NKPC) models (see Galí and Gertler, 1999), is legitimate as such expectations are then unbiased.

Given these powerful results, why should we worry about imperfect knowledge? The modelling strategy they imply is clear: estimate the conditional expectation for the information available. So why is there a problem with current approaches? The following quotes show some of the disconnects from reality:

> It would be an understatement to say that economic forecasts are a constant disappointment to investors. The trouble arises because the forecasters' models are fundamentally flawed. Many are over-reliant on extrapolations of the recent past, while the so-called New Keynesian models deployed by professional economists rarely pick up big economic shifts, such as the 1970s oil shock or the rapid rise of China. Such shifts are inherently unpredictable. John Plender, *Financial Times*, April 22 2017.
> See https://www.ft.com/content/d46942ea-24f5-11e7-8691-d5f7e0cd0a16

During a visit to LSE in 2009, Queen Elizabeth II asked Luis Garicano "why did no one see the credit crisis coming?" to which a part of his answer should have been that DSGE models dominated economic agencies and essentially ruled out such major financial crises by assuming away imperfect knowledge.

Prakash Loungani (2001) argued "The record of failure to predict recessions is virtually unblemished."

How could these dismal outcomes happen given theorems **[1]**–**[5]**? Because imperfect knowledge can have profound consequences for macroeconomic theories, econometric modelling, forecast failure and economic policy.

3   Uncertainty and unpredictability

Uncertainty is ubiquitous in the world and in our knowledge thereof. We are uncertain about many aspects of how economies function; about the accuracy of available data; and about how to model the empirical evidence to reduce our uncertainty. Unpredictability is irreducible uncertainty. Some aspects of unpredictability are measurable and quantifiable in reasonable ways: probabilities can be assigned to represent that unpredictability, as in rolling fair dice. Some events are so unpredictable that reasonable probabilities cannot be assigned, as highlighted by Knight (1921) and by Keynes (1936) 'animal spirits'.

More formally, a random variable X is unpredictable with respect to some information $\mathcal{I}$, if knowing $\mathcal{I}$ does not change knowledge about X. Thus, the distribution $\mathsf{D}_X(X)$ of X is unaffected by knowing $\mathcal{I}$, which entails that the conditional distribution $\mathsf{D}_{X|\mathcal{I}}(X|\mathcal{I}) = \mathsf{D}_X(X)$. There are three levels of unpredictability, dependent on the state of nature and our knowledge thereof:
(a) intrinsic unpredictability about an outcome in a known distribution;
(b) instance unpredictability about an outcome from a 'fat-tailed' distribution;
(c) extrinsic unpredictability about an outcome affected by an unanticipated shift of its distribution.

(a) The Normal distribution is often the basis for probability calculations, as in 'random sampling' from its distribution. Such random variables are 'unknown knowns', where variation derives from chance distribution sampling as with independent errors in statistical theory, which underpin much statistical inference, or random numbers in a simulation. Even so, imperfect knowledge about which draw will arise matters: if you bet on Red but get Black at Roulette... This variety is called *intrinsic unpredictability* because it is a property of the random variable, and is illustrated in Figure 1.

Generalizing to a stationary stochastic process denoted $\{y_t\}$, $t = 1, \ldots, T$, where its DGP is:

$$y_t = f\left(\mathcal{I}_{t-1}\right) + \nu_t \tag{1}$$

when the distribution of $\{\nu_t\}$ is unknown other than $\mathsf{E}[\nu_t|\mathcal{I}_{t-1}] = 0$, where the function $f(\cdot)$ is unknown, and the investigator only has the information set $\mathcal{J}_{t-1} \subset \mathcal{I}_{t-1}$ available, so has imperfect knowledge of
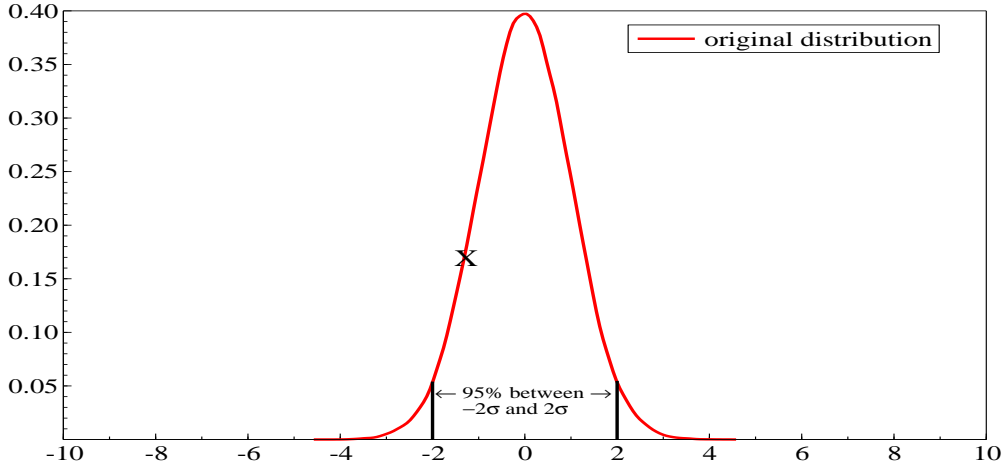
Figure 1: Illustrating intrinsic unpredictability, where σ denotes the standard deviation.

the determinants of the process. Nevertheless, as ν_t is unpredictable from the full information set $\mathcal{I}_{t-1}$, it must be unpredictable from a subset thereof, so:

$$\mathsf{E}\left[\nu_t \mid \mathcal{J}_{t-1}\right] = 0 \tag{2}$$

and hence taking conditional expectations on both sides:

$$\mathsf{E}\left[y_t \mid \mathcal{J}_{t-1}\right] = \mathsf{E}\left[f\left(\mathcal{I}_{t-1}\right) \mid \mathcal{J}_{t-1}\right] + \mathsf{E}\left[\nu_t \mid \mathcal{J}_{t-1}\right] = g\left(\mathcal{J}_{t-1}\right) \tag{3}$$

for some entailed function $g(\cdot)$. Next, let $e_t = y_t - g\left(\mathcal{J}_{t-1}\right)$, then from (3):

$$\mathsf{E}\left[e_t \mid \mathcal{J}_{t-1}\right] = 0 \tag{4}$$

so e_t is a martingale difference with respect to $\mathcal{J}_{t-1}$. Thus predictions of y_t from $g(\mathcal{J}_{t-1})$ are unbiased despite the reduced information set. However, as $e_t = \nu_t + f\left(\mathcal{I}_{t-1}\right) - g\left(\mathcal{J}_{t-1}\right)$, then e_t is not an innovation with respect to $\mathcal{I}_{t-1}$ so predictions will be less accurate (see e.g., Clements and Hendry, 2005).

(b) Instance unpredictability, or 'known unknowns', is illustrated in Figure 2 by outliers from a known 'fat-tailed' distribution, which can occur at unanticipated times, signs, and magnitudes, potentially at 6–8 standard deviations (σs) from the mean–see e.g., Taleb (2007), who refers to these as 'black swan events'. However, several such extreme draws of the same sign are highly unlikely with independent sampling, despite remarks during the Financial Crisis by investment managers complaining about a number of successive events at many standard deviations from the mean of past behaviour.

(c) The most pernicious form is extrinsic unpredictability, or 'unknown unknowns', illustrated by a location shift in Figure 3, where the mean of the distribution changes from a previous 'level' by an unforeseen magnitude and sign at an unanticipated time—as in Soros (2008). Unknown numbers, signs, magnitudes and timings of such shifts can, and do, occur empirically as we will shortly illustrate, as do other forms of changes in distributions. Location shifts make outcomes seem unusual relative to the past, but define a 'new ordinary' situation. If it is not realised the shift has occurred, most draws will seem to be many standard deviations outside what was anticipated from the original distribution, explaining the puzzle for investment managers. As can be seen in Figure 3, it is irrational to hold 'rational expectations'
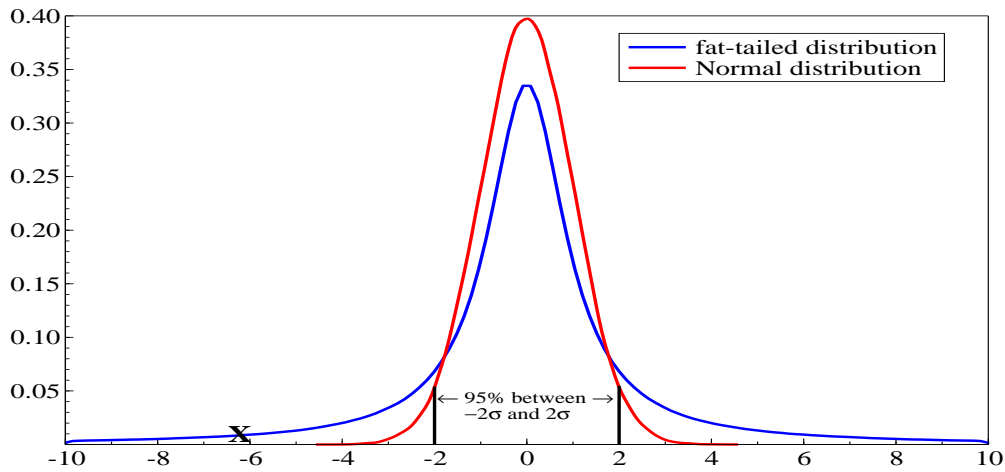
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Figure 2: Illustrating instance unpredictability, where **X** denotes a specific outcome.

based on the current mean when shifts occur: extrinsic unpredictability wrecks economic agents' ability to plan inter-temporally, and leads to forecast failure. Imperfect knowledge about past, present and future shifts has major implications for macroeconomic theory and econometric modelling; policy analyses; and forecasting respectively, issues to which we return below.
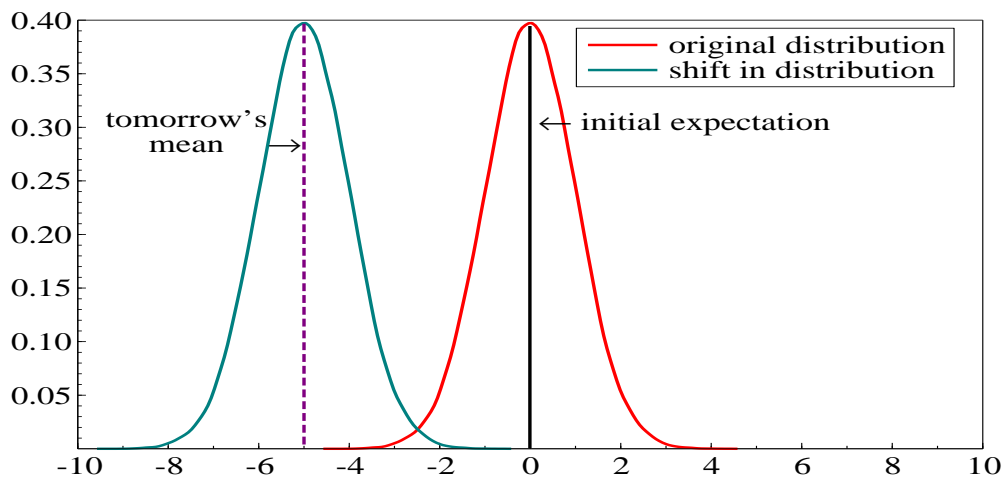

Figure 3: Illustrating extrinsic unpredictability and a location shift.

4 Empirical location shifts over 1860–2016

Hendry (2015) records 55 major UK and world historical events, including 11 dramatic shifts, 6 key financial innovations and changes in credit rationing, 16 important societal changes, 19 salient technology and medical advances, and 9 policy regime shifts.[2] Until they happened, many were unanticipated, and indeed many were thought to be impossible—extreme examples of imperfect knowledge. As Douglas Adams (2002), humorously remarks in *The Salmon of Doubt*:

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[2]See http://www.timberlake.co.uk/macroeconometrics.html

> Oddly, the industry that is the primary engine of this incredible pace of change—the computer industry—turns out to be rather bad at predicting the future itself. There are two things in particular that it failed to see: one was the coming of the Internet, ...;
> the other was the end of the century.

No matter how forward looking economic agents are when planning, radical adjustments to those plans would have been required intermittently. Barro (2009) confirms the high costs of 'consumption disasters', primarily from wars. Figures 4–6 show marked shifts in the time series of variables from economics (post-war UK real GDP growth), society (annual changes in the UK population) and anthropogenic climate influences (UK CO_2 emissions per capita).
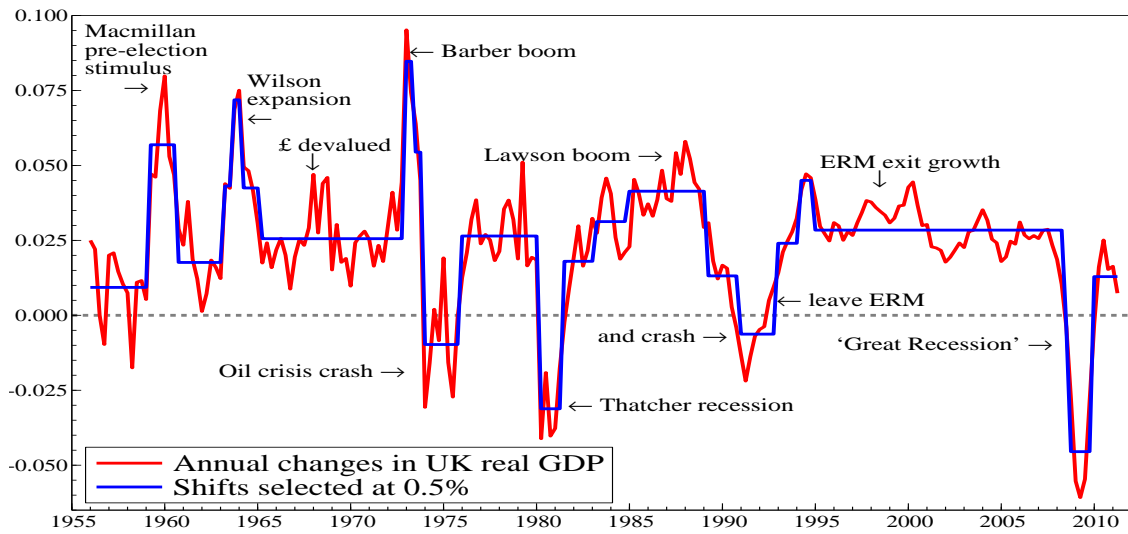

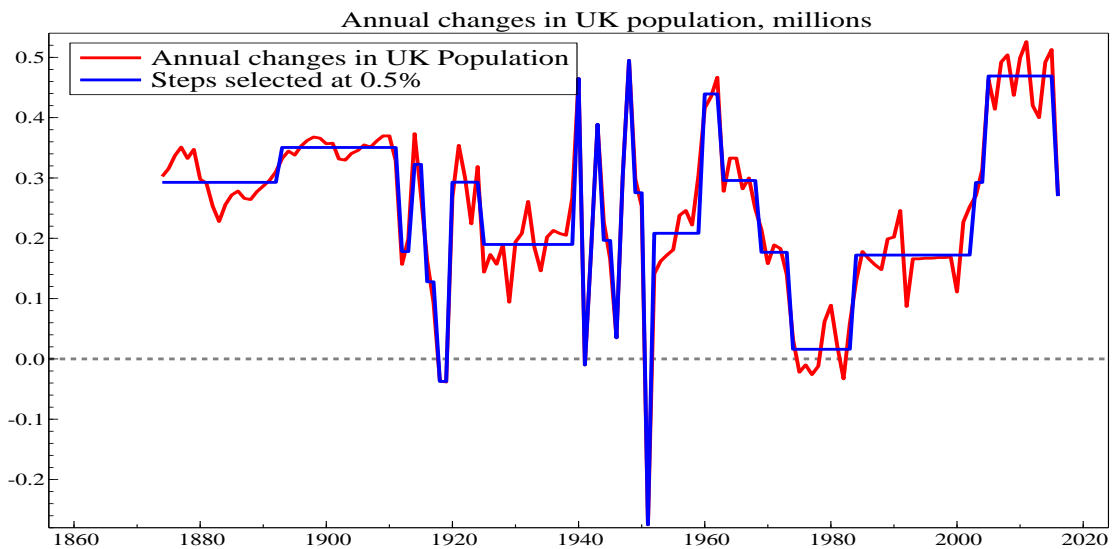Figure 4: Many shifts in post-war UK real GDP growth

Figure 5: Annual changes in the UK population over 1870–2016, in millions

Figure 4 shows that most relatively sudden shifts in UK GDP corresponded to major changes in economic policy–almost always unexpected–but also shows that even a growth rate can be far from a

stationary process. Figure 5 exhibits considerable variation and long swings in changes in UK population over 1870–2016, due to wars, disease, and interactions between birth rates (which fell from 100,000 to 65,000 p.a. over 1964 to 1977, after the availability of the contraceptive pill), death rates, which rose from 1960 to 1980 and since have fallen, and net migration, an issue in may countries. Figure 6 reports per capita UK CO_2 emissions, which rose considerably till 1916, fluctuated violently till 1950, and have dropped dramatically since 1970 (see Hendry, 2017b).
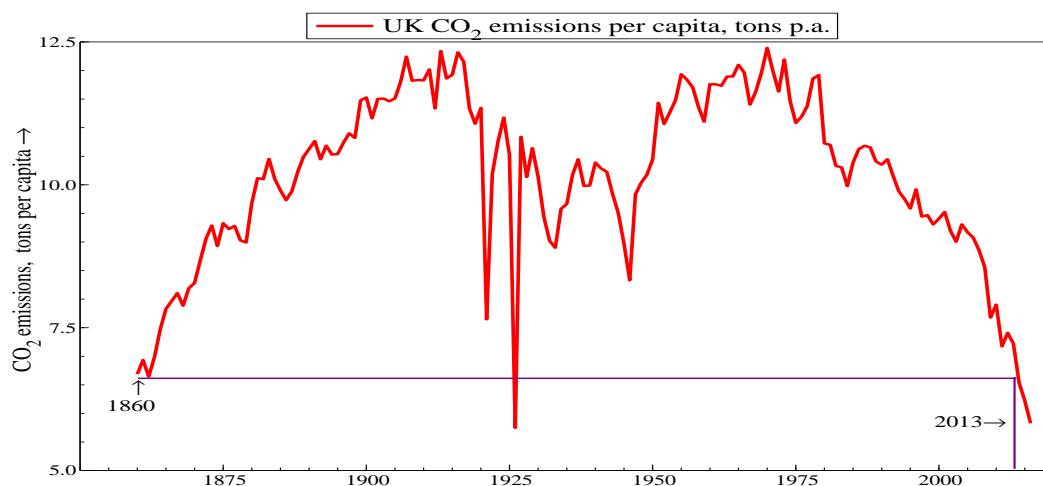

Figure 6: UK CO_2 emissions per capita, in tons per annum over 1860–2016.

All these figures exhibit many major changes: the first two graph step shifts selected at a 0.5% significance level by step-indicator saturation (see Castle, Doornik, Hendry, and Pretis, 2015), so for 150 observations, on average at most one step should be selected by chance under the null (Section 6 summarises why saturation methods can successfully discover the main outliers and location shifts). The sub-period distributions of UK CO_2 emissions in Figure 7 illustrate their changes in shape, spread and location.

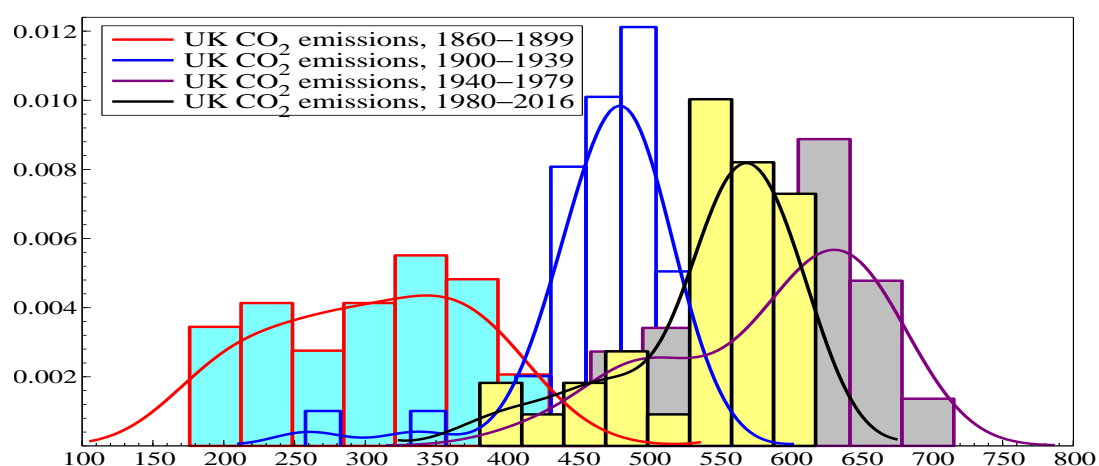Figure 7: Sub-period distributions of total UK CO_2 emissions in millions of tonnes (Mt) pa.

An obvious implication is that any empirical models that ignore sudden large unanticipated shifts will fail to explain the evidence, and indeed will themselves fail at the next large shift. The Financial Crisis and Great Recession exemplify this all too well. The chart in Figure 8 of annual GDP growth
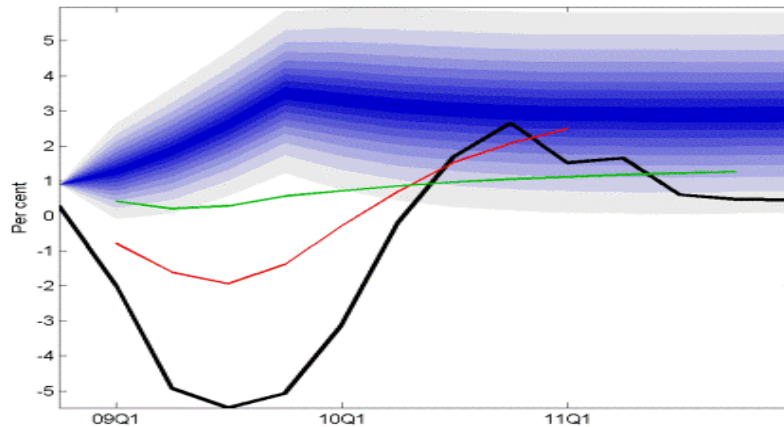
Figure 8: Bank of England ex post COMPASS density GDP-growth 'forecasts' over the Great Recession in blue (from Fawcett, Masolo, Koerber, and Waldron, 2015) and Statistical Suite forecasts in green. Legend: ONS data in black and *Inflation Report* forecasts in red.

forecasts from the Bank of England's COMPASS model over the Great Recession, reveals how a rigid theory-based DSGE performed: the compass was obviously pointing in the wrong direction, as analyzed in Hendry and Muellbauer (2017). Not that the Bank of England was alone. To highlight the problem, Figure 9 shows sequential quarterly forecasts of US GDP at annual rates from 2000(1) to 2011(1) for 1 through 8 steps ahead from an autoregressive model, often used for forecast comparisons.
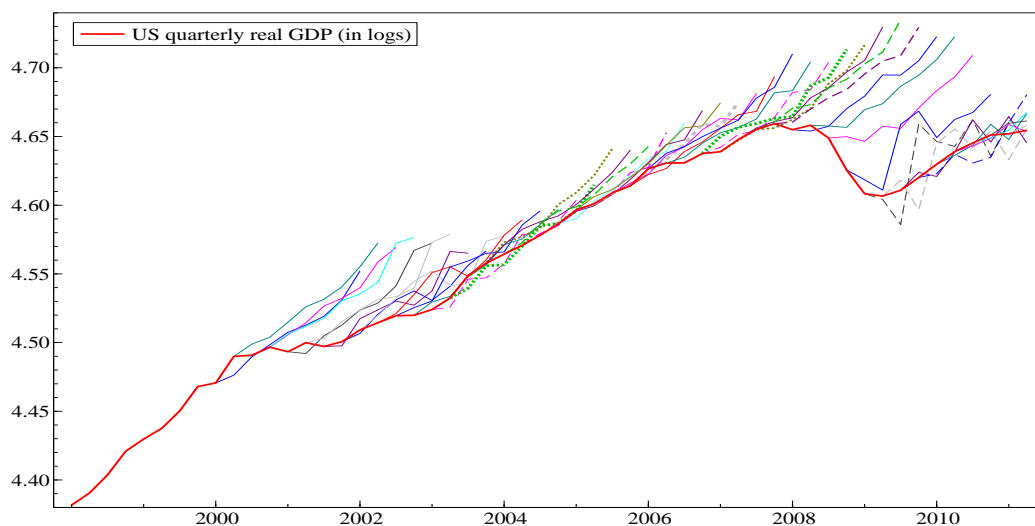
Figure 9: Autoregressive forecasts of US GDP also go wrong over the Great Recession.
Source: Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate. GDPC1 from Federal Reserve Economic Data http://research.stlouisfed.org/fred2.

In a stationary world, forecasts would be about as accurate as their model's in-sample fit: clearly that is not the case in Figures 8 and 9. Intermittent shifts of the kind shown in Figure 3 lead to mis-forecasting, and importantly, the resulting forecast errors are systematic. This arises because most econometric forecasting systems are members of the equilibrium-correction class, including DSGEs and autoregressive models. Because such models embed equilibria, they converge towards them, *irrespective*

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of shifts therein—until either re-estimation eliminates the equilibrium by moving towards a unit-root, or corrects for the shift, or the model 'breaks down' and is replaced, as happened with the Bank of England's earlier BEQEM model. Indeed, the performance of COMPASS and the above US GDP forecasts showing *increases* when output was falling sharply was predictable from the theory of economic forecasting as explained in Castle, Fawcett, and Hendry (2010). The same theory of forecasting predicts that when growth rates are the variable being forecast, forecasts will eventually come back on track, as seen in Figure 7, but will only do so for levels after re-estimation leads to a (near) unit root, or following adjustment (as by intercept corrections: see Clements and Hendry, 1999).

The second main implication is the need for appropriate statistical tools to study evidence from a world that has changed so often, so dramatically–and so often unexpectedly. We consider such tools in Section 6, after considering why the five famous theorems about conditional expectations are mathematically invalid in a world of shifting distributions.

5 Imperfect knowledge: deconstructing the 5 theorems about conditional expectations

The story so far. We noted five famous theorems about conditional expectations, then considered three forms of unpredictability, of which extrinsic unpredictability, particularly from location shifts, was the most pernicious. Next we illustrated the prevalence of distributional shifts empirically, many deriving from major events. Finally, we recorded systematic forecast failures and noted their explanation from shifts of distributions. This leads us to reconsider the empirical validity of the assumptions under which the theorems in Section 3 were proved.

In Euclidean Geometry, the angles of a triangle add to 180^0–a famous theorem proved by generations of school children. Yet if you draw a triangle on a globe and add its angles, the sum is not 180^0. Theorems need assumptions, and Euclid assumed a flat surface. But a globe is not flat, so the theorem is misleading outside its original context, despite a long-held belief that it was 'true' till Bernhard Riemann introduced other geometries.

The five theorems about conditional expectations require that the distributions involved stay constant: and we have just seen numerous 'real world' examples where that assumption is false. For inter-temporal calculations, all five conditional expectations theorems fail when distributions shift. *Imperfect knowledge about shifts has deleterious consequences*.

5.1 Deconstructing rational expectations (RE)

[1] So called 'rational expectations' denote the conditional expectation given available information:

$$y_{T+1}^{re} = \mathsf{E}\left[y_{T+1} \mid \mathcal{I}_T\right] = \mu_{T+1} \tag{5}$$

Such an expression implicitly assumes that the agents forming the expectation know $\mathcal{I}_T$, know how $\mathcal{I}_T$ enters $\mathsf{E}\left[y_{T+1}|\mathcal{I}_T\right]$, and what the relevant distribution, $\mathsf{f}_{y_{T+1}}(\cdot)$, is to integrate over. So far, expectations have simply been written as $\mathsf{E}[\cdot]$, implicitly assuming stationarity, but to clarify what y_{T+1}^{re} really is when distributions can shift, $\mathsf{E}[\cdot]$ needs to be indexed by the relevant distribution, so (5) should be written as:

$$y_{T+1}^{re} = \mathsf{E}_{\mathsf{f}_{y_{T+1}}}\left[y_{T+1} \mid \mathcal{I}_T\right] = \int y_{T+1}\mathsf{f}_{y_{T+1}}\left(y_{T+1}|\mathcal{I}_T\right)\mathrm{d}y_{T+1} \tag{6}$$

Unfortunately, (6) requires a crystal ball for the future conditional distribution $\mathsf{f}_{y_{T+1}}(y_{T+1}|\mathcal{I}_T)$. When there is no time invariance, agents cannot *know* the future $\mathsf{f}_{y_{T+1}}(\cdot)$, or the new conditioning relation

between y_{T+1} and $\mathcal{I}_T$, so the best they can do is form some sort of 'sensible expectation', y_{T+1}^e by estimating (guessing?) the next period's distribution $\widehat{f}_{y_{T+1}}(\cdot)$ leading to:

$$y_{T+1}^e = \int y_{T+1} \widehat{f}_{y_{T+1}} \left(y_{T+1} | \mathcal{I}_T \right) \mathrm{d} y_{T+1} \tag{7}$$

If $f_{y_{T+1}}(y_{T+1}|\mathcal{I}_T)$ changes unexpectedly, there are no good rules for forming $\widehat{f}_{y_{T+1}}(\cdot)$, but assuming that $\widehat{f}_{y_{T+1}}(\cdot) = f_{y_T}(\cdot)$ is not a good decision when $\mathsf{E}_{f_{y_T}}[y_{T+1}|\mathcal{I}_T] = \mu_T \neq \mu_{T+1}$ as Figure 3 illustrated. Inter-temporal conditional expectations are biased when location shifts occur: theorem **[1]** is false in a real-world setting.

Naturally, one would like to know how to form a useful expectation, but if the shift is genuinely unanticipated, there cannot be a good way. Nevertheless, once the shift has happened, there are several possible strategies. The first is to ignore its occurrence, as DSGEs tend to do, since they assume it could not have happened. This will lead to systematic forecast failure as in Figures 8 and 9 after the Financial Crisis, since equilibrium-correction models converge back to the old, and now incorrect, equilibrium. The alternative is to adjust to the new location following the shift, either immediately or partially, to correct the previous forecast error, using what Castle, Clements, and Hendry (2015) call robust forecasting devices. If agents sensibly adopt the second route, then econometric models must embody that behaviour and abandon the pretence of 'rational expectations' with no error-correction mechanisms.

5.2 Inter-temporal law of iterated expectations

It will not be a surprise that theorem **[2]** now also fails facing distributional shifts. To understand why it worked above, as that reveals when it will not hold, consider variables, (y_{T+1}, y_T), at successive dates that are drawn from the same time-invariant distribution $f = f_{y_T}(\cdot) = f_{y_{T+1}}(\cdot)$ so that:

$$\mathsf{E}_{f_{y_T}} \left[\mathsf{E}_{f_{y_T}} \left[y_{T+1} \mid y_t \right] \right] = \mathsf{E}_{f_{y_{T+1}}} \left[y_{T+1} \right] \tag{8}$$

which is proved as follows:

$$\mathsf{E}_{f_{y_T}} \left[\mathsf{E}_{f_{y_T}} \left[y_{T+1} \mid y_T \right] \right] = \int_{y_T} \left(\int_{y_{T+1}} y_{T+1} f_{y_T} \left(y_{T+1} | y_T \right) \mathrm{d} y_{T+1} \right) f_{y_T} \left(y_T \right) \mathrm{d} y_T$$

$$[\textbf{collect terms}] = \int_{y_T} \int_{y_{T+1}} y_{T+1} f_{y_T} \left(y_{T+1} | y_T \right) f_{y_T} \left(y_T \right) \mathrm{d} y_T \mathrm{d} y_{T+1}$$

$$[\textbf{p(a,b)=p(a|b)p(b)}] = \int_{y_{T+1}} y_{T+1} \left(\int_{y_T} f_{y_T} \left(y_{T+1}, y_T \right) \mathrm{d} y_T \right) \mathrm{d} y_{T+1}$$

$$[\textbf{integrate out } y_T] = \int_{y_{T+1}} y_{T+1} f_{y_T} \left(y_{T+1} \right) \mathrm{d} y_{T+1}$$

$$[\textbf{constancy of } \mathsf{f}(\cdot)] = \int_{y_{T+1}} y_{T+1} f_{y_{T+1}} \left(y_{T+1} \right) \mathrm{d} y_{T+1}$$

$$= \mathsf{E}_{f_{y_{T+1}}} \left[y_{T+1} \right] \tag{9}$$

The law of iterated expectations holds because $f_{y_T}(\cdot) = f_{y_{T+1}}(\cdot)$, so fails when the distribution shifts: then the substitution on the second last line is invalid because $f_{y_T}(\cdot) \neq f_{y_{T+1}}(\cdot)$. Hendry and Mizon (2014) provide formal proofs. Inter-temporal derivations are invalidated by unanticipated shifts, so either theory-models that use such analyses require the absence of distributional shifts, so are empirically irrelevant, or–as with DSGEs–are intrinsically non-structural because their mathematical basis fails when distributions alter. Since almost all economic policy changes involve location shifts, often unanticipated, economists cannot rely on theory-based model selection alone.

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The invalidity of theorems **[3]**–**[5]** follows inexorably: not knowing the shift as part of the conditioning information will produce biased expectations; mis-specifications due to unmodelled location shifts will produce biased forecasts; and it is inappropriate to replace unknown expectations by realized future outcomes as shown in Castle, Doornik, Hendry, and Nymoen (2014). Imperfect knowledge is potentially disastrous for a wide range of economic theories and models: but empirical modelling can be fruitfully undertaken.

6   Efficient empirical modelling with tools to detect shifts

Impulse-indicator saturation creates an indicator for every observation, where the indicator is zero everywhere except for unity at that observation, denoted $1_{t=j}, j = 1, \ldots, T$ so there are T such indicators for T observations. I discovered a way to handle as many, or more, candidate variables than observations in Hendry (1999) by first including in my model a block of such indicators for the first half of my sample (a way to conduct a Chow (1960) test proposed by Salkever, 1976), then dropping the insignificant indicators and adding the block for the second half.[3]

To understand how indicator saturation methods are able to detect shifts and changes, first consider a setting where your model is correctly specified other than a single shift, but you know where that shift occurs. Split your data at that point and fit your model separately to each partition. Then you would be very surprised if doing so did not deliver the appropriate sub-sample estimates of the parameters. Similarly, if you knew where each of several shifts had occurred and split your sample accordingly, or introduced the correct form of indicator function to capture the elements that were changed by the shift but kept all other parameters unchanged. Seeking the split by indicator saturation, which will always include the correct indicator, may add some variability, but should select the correct one or one close to it, and thereby accomplish the same task. Of course, small shifts will not be detected, but then again their omission should not be too harmful. Importantly, as there may be several shifts, a multiple block search algorithm like *Autometrics* (or an R version as in Pretis, Reade, and Sucarrat, 2017) is essential, as finding any one shift may depend on finding others.

This idea has led to methods for also selecting lag lengths and non-linearities combined with tackling multiple location shifts and outliers of unknown numbers, timings, signs and magnitudes, all jointly–essential when finding some effects depends on finding others–implemented in *Autometrics*, our powerful computational tool (see Doornik, 2009). Searching for outliers and breaks can usually be accomplished while retaining all other candidate regressor variables. I and co-authors have published extensively on why such an approach works so well and so should always be adopted, being vastly preferable to computing 'hundreds of regressions' as in Friedman and Schwartz (1982) then selecting those one 'liked', a practice earlier criticised by Leamer (1983) and Spanos (2000) since genuine significance cannot be controlled. Instead, Hendry and Doornik (2014), Hendry and Johansen (2015) and Hendry (2017a) show how a theory model can be retained unaffected by selecting only over other orthogonalized candidate variables, such that when the theory model is correct, the distributions of its parameter estimates are *identical* to those obtained from directly fitting that model to the data, so nothing is lost. However, when the theory model is incomplete, incorrect, or changes over time, an improved model results from selecting over those additional variables.[4]

Analyses and applications of indicator saturation methods include Hendry, Johansen, and Santos (2008), Johansen and Nielsen (2009, 2013, 2015), Marczak and Proietti (2016) and Ericsson (2017) for impulse-indicator saturation (IIS); Castle, Doornik, Hendry, and Pretis (2015) for step-indicator saturation (SIS); Ericsson and Reisman (2012) for super saturation, where IIS is undertaken jointly with SIS;

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[3]See http://voxeu.org/article/data-mining-more-variables-observations for an explanation.

[4]See http://voxeu.org/article/improved-approach-empirical-modelling-0 for an explanation and application.

Ericsson (2012), Kitov and Tabor (2015) and Castle, Hendry, and Martinez (2017) for multiplicative-indicator saturation which interacts step indicators with variables; and Pretis, Schneider, Smerdon, and Hendry (2016) and Schneider, Smerdon, Pretis, Hartl-Meier, and Esper (2017) for the concept of 'designer break functions', there applied to detecting the temperature impacts of volcanic eruptions.

Thus, despite incomplete and imperfect knowledge about the causal mechanisms hidden in a welter of information, and about past location shifts, empirically congruent models can be selected while retaining theory insights, even when there are more candidate variables than observations. However, even detecting in-sample shifts does not rescue either the law of iterated expectations or models reliant thereon, nor does it ensure any resulting models will not fail from a future unanticpated location shift. As noted above, robust forecasting devices can help avoid systematic forecast failure despite even *ex post* imperfect information as to the causes of any shift.

7 Conclusions

Imperfect knowledge about the world and how it functions is ubiquitous–but some aspects of ignorance are more troublesome than others. In particular, lack of knowledge about shifts of distributions can lead to model break down, forecast failure and incorrect policy responses, as well as mistaken decisions by individuals. This conclusion was reached by first explaining powerful theorems about the properties of conditional expectations, which we later show depended on the distributions involved remaining constant. Next, the three forms of unpredictability (intrinsic, instance and extrinsic) highlighted the consequences of distributions shifting unexpectedly, so 'today's mean' would be a poor predictor of 'tomorrow's mean'. Then, we exhibited examples of empirical shifts across economics, society and anthropogenic CO_2 emissions, and illustrated the implications for forecast failure, widely experienced during the Financial Crisis and ensuing Great Recession. Consequently, extrinsic unpredictability in the guise of imperfect knowledge about location shifts entails that 'rational expectations' can be irrational, and the mathematical basis of DSGEs is invalid when such shifts occur. Nevertheless, it is feasible to successfully empirically model ever-changing worlds using indicator saturation methods to correct for shifts in-sample, and robust methods for forecasting out-of-sample after shifts.

There is a further conclusion, so far not alluded to, concerning the uses and mis-uses of mathematics in economics, discussed in Hendry (2011). Without careful mathematical analysis, the flaws in applications of inter-temporal optimization and claims about rational expectations investigated in Section 5, could not have been revealed. Mathematical analysis is also essential to understand economic forecasting and derive general implications, several of which were used in Section 4, including why forecasts rise when data falls as in Figures 8 and 9. Finally, an econometric theory of model selection facing unknown numbers of shifts and with more candidate explanatory variables than observations, as discussed in Section 6, would have been impossible without mathematical and statistical tools. The complaints about the roles of mathematics in economics should focus on its mis-use, and on inappropriate and inadequate formulations, not on its use.

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