Method to simultaneously determine stock, flow, and parameter values in large stock flow consistent models

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Abstract

Stock flow consistent macroeconomic models suffer from the lack of a coherent estimation method due to the complicated nature of the modeling process. This paper provides a candidate estimation method that determines the values of each stock and flow simultaneously by analytically solving any stock flow model, and converting the estimation into a global minimisation problem in $p - k$ dimensions. We describe the method and apply it to a canonical model using real-world data. The method estimates the parameters and flows reliably.

Keywords: Instability, finance, estimation, stock flow consistent models.
JEL Codes: E32; E37; E51; G33.

1 Motivation

The core of macroeconomics is the empirical and statistical description of the behavior of aggregations of economic actors. Stock flow consistent macroeconomic modeling emphasizes the connections between classes (or sectors) of economic agents.

Until now, as far as we are aware, no explicit method has been given for the solution and estimation of stock flow models. Our goal in this paper is to provide such a method.

In stock flow consistent models the economy is treated as a set of sectors interacting with one another, for example: households, firms, private banks, the central bank, and the rest of the world. In each sector, say, households,

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they buy from firms, the firms sell to the households, netting out to zero at any moment in time. The sectors are tied together within a balance sheet for the economy, and their transactions recorded within transactions flow matrices and revaluation matrices for capital gains.

Every flow and every stock variable is logically integrated into the accounting so that the value of any one item is implied by the values of all the others taken together; in other words the system of accounts is stock-flow consistent .

The model is written out as (sometimes hundreds of) balancing and identity equations, with, for example, the amount of consumption, C, demanded by households C_d equal to the amount of consumption supplied by firms, C_s. So it goes for wage bills, investment in capital goods, bonds issued by banks to firms and households respectively, and so on.

Next come the behavioral equations. Here we care about how much consumption will increase when disposable income increases, and what proportion of the increase in consumption will come from current income, and how much from past wealth.

Finally there are equations to revalue capital gains and losses, and the model is closed with a 'hidden' or redundant equation whose values are implied by the existence of all of the other equations in the model.

Most equations are linear, so stock flow consistent models can normally be solved for their steady state, and the behavior of the entire system can be simulated. Choosing stock-flow norms, which must be stable, to make the simulated models work, is a serious concern at this stage, and each of these models will require attention to the choice of initial conditions for parameter values.

The simulated system is then shocked, via a drop in investment, say, or a change in wages, or a change in inflation, and the behavior of the system can be analyzed and discussed by comparison with a baseline or series of baselines. There is a burgeoning research community for these types of models. Stock flow consistent models also naturally model, amongst others, the distinction between wage earners and the recipients of capital income van Treeck (2009), financial imbalances Godley and Izurieta (2004), contagion effects, Kinsella and Khalil (2011) and as well as income distribution effects Dos Santos and Zezza (2008).

Finding stock flow norms is, at present, a black art, and more error than trial is involved in finding them as Taylor (2008) argues. This is unsatisfactory intellectually, but also raises a practical concern over the stability of these models. If they are sensitive to small changes in the values of simple parameters like the propensity to consume out of past income by households, say, then how valid are they as representations of reality?

The contribution of our paper, and indeed the effort of our research program, is to provide the missing link between the simulated worlds described by Godley and Lavoie to a coherently estimated model built from real world data.

The paper is laid out as follows. Section 2 lays out in general terms the problem we face and an algorithm to find the solution of a Stock Flow Consistent model. Section 4 provides a practical application of the method using a canonical model, the INSOUT model developed by Godley and Lavoie (2007). Section 5 concludes with directions for future research.

2 Theory

Any stock flow consistent model can be represented in the following matrix form:

$$\begin{bmatrix} F(t) \\ S(t) \end{bmatrix} = A(t) \begin{bmatrix} F(t-1) \\ S(t-1) \end{bmatrix} \tag{1}$$

In equation 1, $S(t)$ is a vector of dimension n containing all the stock values at time t, and $F(t) = C(t)S(t-1)$ i an m-dimensional vector containing all the flows in time t with $m \geq n$. $A(t)$ is a square parameter matrix of dimension $n+m$. The parameters vary with time as they represent behavioral equations. The values of $S(t)$ and $S(t-1)$ can be assumed known as they come from nationally produced flow of funds accounts, with quartertly frequency in many cases.

In general then, we can take the stock values $S(t, t+1, \ldots)$ as known in each period. However let us assume both flows and parameters are unknown[1]. Given the number m of flows and the number p of parameters to find and the number $m+n$ of equations, we clearly face an undetermined system of equations. Often there are k constraints on the parameters (for example, when estimating portfolio choice equations of Tobin-Brainard form). We end up with $p-k$ independent parameters and n flows to determine simultaneously.

2.1 Sketch of the idea

The methodology proposed here is based on a very simple idea: determine the flows and the parameters simultaneously such that:

- the observed variation in stocks is respected (i.e the predicted change in wealth for the model, given the values of the parameters and the flows is equal to the observed one).
- each parameter lies in a economically meaningful interval (for example, perhaps we have reason to think that the propensity to consume out of income lies between 0.4 and 0.9 rather than 0.0001).
- the absolute value between observed GDP and predicted GDP (or another equivalent quantity of interest) is minimized.

2.2 Proposed method

Here are the following steps of the proposed method:

1. For a given model known to be stock-flow consistent;

[1] Please note that the flows may be found in the statistical tables but are often inconsistent with the stocks found in the same tables.

2. Obtain the n flows as a function of the $p-k$ unknown parameters and the $2m$ observed stock values by solving analytically the model, and;

3. Minimize the absolute value between observed GDP and predicted GDP for the $p-k$ parameters in the domain defined by the meaningful intervals on the remaining $p-k$ parameters.

We thus face a global minimisation problem in $p-k$ dimensions.

2.3 Algorithm

Given the complexity of the minimisation and the computational difficulties to find a global minimum for a more than quadratic function, we propose the following algorithm to find a minimum which might be local.

1. For all exogenous variables (and GDP):
    (a) Select the equations determining the variable X;
    (b) Replace in each equation the known values (present and previous stocks, previous flows[2], previous parameters, already estimated parameters);
    (c) Solve the system for the parameters present in the equations;
    (d) Find the parameters that minimise the distance between predicted value and the observed value of X under constraints, if any, for the parameters, in the domain defined by the economically meaningful intervals for each parameter.

2. Repeat the operation for the next variable.

2.4 Steady state vs long run vs instability

Stock flow consistent models can been used to fill the gap between short-run and long-run analysis Dos Santos and Macedo e Silva (2009); Ryoo (2010). The long-run has been defined in diverse ways such as:

1. in the long-run capacity utilisation is at its target rate (this is the Harrodian definition);

2. or profit rate meets its target (this is the Kaleckian specification);

3. or in the long run technology may change while it is constant in the short run (this is the structuralist view).

[2]It is important to note that the initial value for the flows, that is $F(0)$ will still have to be estimated. These values will of course influence the parameters' estimation.

It is important to note that the method proposed here is by definition a short run analysis, since all parameters change, and can change period to period. However, we will be able to observe trends (and long or short oscillations) in parameters. Furthermore, some parameters might be more volatile than others.

Our method does not discuss the existence (or otherwise) of a steady state. It might be the case that with the parameters' estimated value, there is no steady state. or that the existing steady state is unstable. We argue however that this is not important as all parameters change in each period. The steady state existence and stability analysis might then be discussed in the light of the value estimated for each parameter and the trends observed.

3 Theoretical example: the SIM model

Model SIM is not a good representation of reality but is a good simple example to use for out methodology. It has the advantage to be very easily tractable, the dynamics are very straightforward to understand and it reduces all computational time to almost nothing. It is thus easy to highlights the different characteristics of the proposed methodology with model SIM.

3.1 Description of the model

Model SIM is the simplest model presented in Godley and Lavoie (2007). The transaction-flow matrix is given by table 1.

	Households	Production	Government	$\sum$
Consumption	$-C$	$+C$		0
Govt. expenditures		$+G$	$-G$	0
Wages	$+WB$	$-WB$		0
[Output]		$[Y]$		
Taxes	$-T$		$+T$	0
Change in money stock	$-\Delta H$		$+\Delta H$	0
$\sum$	0	0	0	0

Table 1: Transaction-flow matrix of SIM model.

The economy is closed and composed of tree sectors: households who receive wages W in exchange of labor, pay taxes $T$and consume C out their disposable income YD; firms who produce an an output Y which is sold to households and the governemnt and pay wages in exchange for labor; and a government which buys output G from the firms and receive taxes from the household sector. There is only one asset: money stock H. All income that is not consumed by household is thus saved as cash. If household have positive savings then the government has to have a deficit. The following equations describe the model[3].

[3]For simplicity we have dropped all the supply equal demand equations and removed all the subscripts referring to demand and supply.

Equation (9) is the hidden equation.

$$YD = W.N - T \tag{2}$$
$$T = \theta.W.N \tag{3}$$
$$C = \alpha_1.YD + \alpha_2.H_{h,-1} \tag{4}$$
$$Y = C + G \tag{5}$$
$$N = \frac{Y}{W} \tag{6}$$
$$\Delta H_s = G - T \tag{7}$$
$$\Delta H_h = YD - C \tag{8}$$
$$\Delta H_h = \Delta H_s \tag{9}$$

The model is thus composed of 6 flow-variables, one stock-variable and 3 parameters: θ the tax rate, α_1 the propensity to consume out of income and α_2 the propensity to consume out of wealth.

3.2 Solution of the model

If we assume the variation in wealth to be observable and that government spending is the exogenous variable, almost all the flows are determined, for a set of values for the parameters. The only flows that cannot be determined are the wage and the employment. However if one these two is fixed, then all flows are determined. The following equations gives the solution where the variables with an $\overline{\text{overline}}$ are variables for which the value is given.

$$T = \overline{G} - \overline{\Delta H_s} \tag{10}$$
$$Y = \frac{\overline{G} - \overline{\Delta H_s}}{\theta} \tag{11}$$
$$YD = (1-\theta)\frac{\overline{G} - \overline{\Delta H_s}}{\theta} \tag{12}$$
$$C = \frac{\overline{G} - \overline{\Delta H_s}}{\theta} - \overline{G} \tag{13}$$
$$C = \alpha_1(1-\theta)\frac{\overline{G} - \overline{\Delta H_s}}{\theta} + \alpha_2\overline{H_{h,-1}} \tag{14}$$

It is interesting to note that in order for the model to have a solution, both equation (13) and (14) have to be respected. This creates a constraint on the value that the parameters may take, given the values for the exogenous variables. This constraint is given by (15)

$$\theta = \frac{(1-\alpha_1)(\overline{G} - \overline{\Delta H})}{\overline{H_{-1}}\alpha_2 + \overline{G}(1-\alpha_1) + \alpha_1\overline{\Delta_H}} \tag{15}$$

However, this constraint does not allows to determine all the parameter's value. This constrains us to use the algorithm described in section 2.3 to fix the parameters' value.

3.2.1 Estimation of the parameter

In order to show how the algorithm functions, what are the importance of some characteristics such the economically meaningful domain and what are its limitation, we have run several estimation all with generated data. To generate the data we have created random increase in wealth and in government expenditure. We have then randomly created value for two of the parameters and determined the value of the third one, given the constraint (15). Finally we have generated the flows using equation (10) to (13).

The first experiment we have run is simply to run the algorithm on the generated data where all flows correspond exactly to one set of parameters' value for each period. Since we know the "true" value of the parameters we can then compare the computed value with the "true" ones. The results are presented in figure 1 show these results. We observe that GDP (and all the flows as a matter of fact) are perfectly predicted by the model. More interestingly, the predicted value for the tax rate is exactly equal to the rel one. The result for both propensities is less good. This highlights the fact that these parameters are undetermined given the flows. Indeed, even when the flows are observed without statistical error, there is an infinity of values for each of these two parameters so that equation (4) is respected. In order to avoid such an indetermination, the choice of the domain is essential.

3.3 Choice of the domain

Our next experiment works on the domain choice. As we have seen in the previous section, cases of indetermination exists. In order to restrain the number of solution, we have to carefully chose the domain for each of the parameters. We have run the same experiment as described in the previous section but with different domain size for the propensities to consume.

The results are shown in figure 2. We observe that average relative errors increase as the size of the interval increases. However, the impact is larger for the propensity to consume out of wealth (red line) than for the propensity to consume out of income (blue line). This is due to the fact that the average value for the propensity to consume out of wealth is much smaller than the average propensity to consume out of income (0.2 and 0.6 respectively). An error of 0.2 has thus larger impact on α_2 than on α_1. It is also interesting to note that the error magnitude follow the same trends. Either both errors are small or they are large. The reason for that is fairly obvious: when one propensity to consume if badly estimated, the other has to compensate and thus is also going to be badly estimated.

Figure 1: We observe that GDP (and all the flows as a matter of fact) are perfectly predicted by the model. More interestingly, the predicted value for the tax rate is exactly equal to the rel one. The result for both propensities is less good.

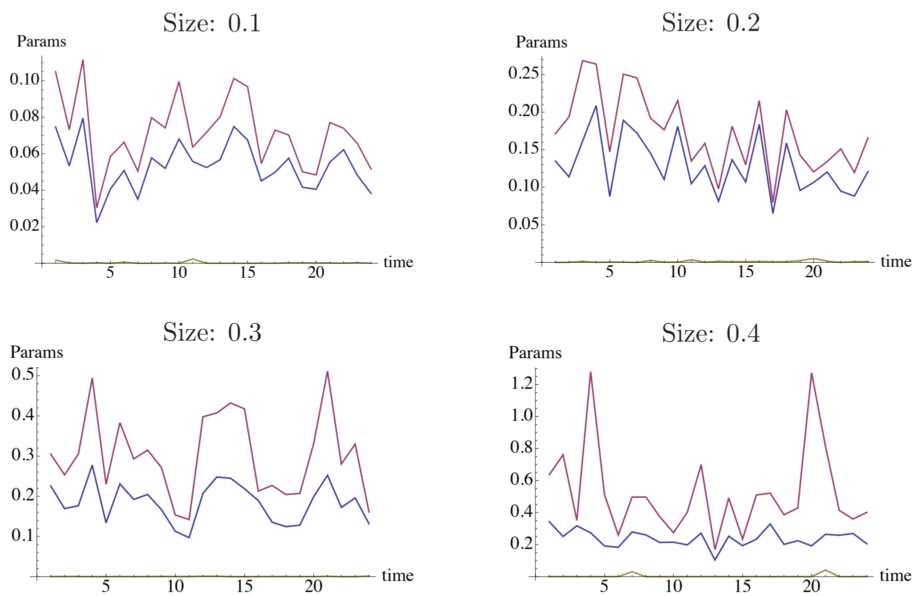

Figure 2: We observe that average absolute relative errors increase as the size of the interval increases. However, the impact is larger for the propensity to consume out of wealth (red line) than for the propensity to consume out of income (blue line).

4 Practical example: the INSOUT model

4.1 Description of the model

The INSOUT model is defined fully in Godley and Lavoie (2007, chapter 10). The model is of reasonable complexity at 53 equations, since it combines inside and outside money in a comprehensive manner. The original purpose of the model was to describe the main ways in which a central bank can exercise control over its commercial banks. There are three types of money in the model: cash, checking accounts M1, and deposit accounts M2. Furthermore, the model is the book's first attempt to model describe portfolio choice, following Tobin's approach. We propose a simplified version of it without reserves and where checking deposits and high powered money have been aggregated.

We deliberately chose a model of intermediate complexity[4] to illustrate the applicability of the method to larger stock flow consistent models, while retaining enough simplicity to be 'legible' for the reader.

In the model every column sums to zero, and then it follows that once every variable in a column bar one has been determined that last variable is logically implicit. This logical constraint on the sum of a sector's activities has a causal interpretation, because, with all decisions having to be made in an uncertain world, there has to be, for every sector, some component of the sum of their transactions which flexibly takes on the character of a residual, and which, as Godley and Lavoie emphasize, cannot be directly controlled[5].

For households the residual process will be mainly the way in which their holdings of non-interest bearing credit money (checking accounts) change; for firms the residual will be the amount of loans still due to the banking system; for banks it will be holdings of bills and on occasion the advances that they take from the central bank; for the government, it will be new issues of bills; for the central bank, it will be the issue of base, or high-powered, money to banks, as well as the amount of its advances to commercial banks.

The balance sheet table of our model is given by table 2. A complete description of the adapted model's equations and the transaction matrix is given in Appendix A.

There are five sectors in this model: households, firms, a government, a central bank, and private banks. Households can hold a variety of financial assets: high powered money issued by the central bank H, time deposits issued by banks M, Bills issued by the central bank, B, bonds issued by the government, BL, and at the end of each period, each has a balance of V in its wealth. Firms hold inventories IN and loans L. The government issues bills, bonds, and its balance is the government deficit GD at the end of each period. The central bank issues high powered money and holds government bills. The private banks

[4]The model lacks capital stock and investment behavior. The stock market is not modelled and all profits are distributed.

[5]This approach follows from Foley (1975), where Foley shows how, unless having perfect foresight, it is impossible to have stock and flow equilibrium altogether. However with "buffer stock assets", this results doesn't hold as it is the buffer stock assets that ensure that both equilibrium (flow and stock) are attained.

	Households	Firms	Govt.	Central bank	Banks	Σ
Inventories		+IN				+IN
HPM/deposits	+Hh			-H	+Hb	0
Time deposits	+M				-M	0
Bills	+Bh		-B	+Bcb	+Bb	0
Bonds	+BL pbL		-BL pBL			0
Loans		-L			+L	0
Balance	-V	0	+GD	0	0	-IN
Σ	0	0	0	0	0	0

Table 2: Balance Sheet of INSOUT model.

hold high powered money, issue time deposits, hold bills, and issue loans. The system is stock flow consistent.

4.2 Algorithm

There are 28 parameters with 8 constraints on the portfolio choice parameters, leaving out 20 seemingly independent parameters. As mentioned in the previous section, finding a global minimum for a 20-dimension function is very complex, furthermore the domain based on the economically meaningful intervals for each parameter might render the use of numerical minimisation algorithm highly unstable. We thus follow the algorithm defined in section 2.3.

1. The first variable to be used is GDP, Y.

   (a) We start by finding the system of equations determining nominal

GDP. These equations are:

$$s^e = s_{-1} \tag{s^e}$$
$$in^T = \sigma^T s^e \tag{in^T}$$
$$in_{-1} = \frac{IN_{-1}}{UC_{-1}} \tag{in_{-1}}$$
$$in^e = in_{-1} + (in^T - in_{-1})\gamma \tag{in^e}$$
$$y = in^e - in_{-1} + s^e \tag{y}$$
$$N = \frac{y}{pr} \tag{N}$$
$$W = W_{-1}(1 + \omega_W) \tag{W}$$
$$WB = N.W \tag{WB}$$
$$UC = \frac{WB}{y} \tag{UC}$$
$$IN = in.UC \tag{in}$$
$$in = in_{-1} - s + y \tag{s}$$
$$NHUC = (1 - \sigma^T)UC + \sigma^T UC_{-1}(1 + r_{l-1}) \tag{$NHUC$}$$
$$p = (1 + \tau)(1 + \phi)NHUC \tag{p}$$
$$Y = p.s + UC\Delta in \tag{Y}$$

(b) The set of parameters determining GDP, given previous and current stock values and previous flow values is $\{pr, \omega W, \gamma, \sigma T, \tau, \phi\}$ where pr is the productivity, ωW is the parameter determining te change in wages, σT is the targeted ratio of inventories over sales, τ is the tax rate and ϕ is the mark-up on prices.

(c) We solve the system of equations

(d) We then find the value for these parameters that minimise the distance between predicted an observed GDP.

2. The next variables to be used are government expenditures, G and interest rates on bonds rbL. We use these two variables since the set of parameters to be used have opposite impacts on each variable.

   (a) We start by finding the system of equation determining G and rbL.

   (b) The set of parameters determining G, given previous and current stock values and previous flow values and the values found for $\{pr, \omega W, \gamma, \sigma T, \tau, \phi\}$ is $\{\alpha_1, \alpha_2\}$, the propensity to consume out of income and wealth, respectively.

   (c) We solve the resulting system of equations.

   (d) We then find the value for these parameters that minimise the distance between predicted an observed values for both variables

3. The next variable to be used is interest rate on bills rb.

   (a) We start by finding the system of equation determining rb.

   (b) The set of parameters determining rb, given previous and current stock values and previous flow values and the values found for $\{pr, \omega W, \gamma, \sigma T, \tau, \phi, \alpha_1, \alpha_2\}$ is $\{\lambda_{20}, \lambda_{22}, \lambda_{33}, \lambda_{35}, \lambda_{44}, \lambda_{45}, \lambda_c, \xi_b, \xi_m\}$ where all λ's are portfolio choice parameters, λ_c is the share of consumption hold as cash and ξ_b and ξ_m are updating parameters of the interest rate on loans and deposits respectively.

   (c) We solve the system of equations

   (d) We then find the value for these parameters that minimise the distance between predicted an observed interest rates on bills[6].

4.3 Results

We began with annual data for the US economy from 1985 to 2007. We then generated GDP and government expenditure flows based on randomly generated parameters and observed stock values. We randomnly shocked GDP by a positive or negative 0 to 10% variation. We therefore have a variational system where the parameter values and flows are known before the esimation procedure, but with some error.

Before analysing the results, we need to emphasize the issue of the "indefinitedness" of the parameters. Taking the equations (16), (17) and (18) which determine the targeted inventories to sales ratio, the targeted real wage and the nominal wage respectively. These 3 equations contain 6 parameters. Even when constraining the value that these parameters in economically meaningful intervals, the value that these parameters may take are infinite.

$$\sigma^T = \sigma_0 - \sigma_1 r_l \tag{16}$$

$$\omega^T = \Omega_0 + \Omega_1 pr + \Omega_2 \frac{N}{N_{fe}} \tag{17}$$

$$W = W_{-1}\left(1 + \Omega_3\left(\omega^T - \frac{W_{-1}}{p_{-1}}\right)\right) \tag{18}$$

In order to simplify the estimation phase, we have simplified these "indefinite" parameters either by using σ^T as a parameter and removing equation (16) or by removing equation (17), creating a new parameter ω_W and using equation (18.A) instead of (18). The value of these non estimated parameters may then be obtained ex-post, after having calibrated the model.

$$W = W_{-1}(1 + \omega_W) \tag{18.A}$$

$$\omega_W = \Omega_3\left(\omega^T - \frac{W_{-1}}{p_{-1}}\right) \tag{19}$$

[6] *Mathematica* code for the algorithm is available from the corresponding author upon request.

The algorithm described in section 4.2 was applied and the parameters were estimated. Table 3 gives the intervals used during the algorithm's execution.

Parameter	Symbol	Lower value	Upper value
Wage adjustment	ωW	0	0.07
Inventories adjustment	γ	0.2	0.6
Inventories to sale ratio	σT	0	0.3
Tax rate	τ	0.2	0.5
Mark-up	ϕ	0	0.1
Propensity to consume out of income	α_1	0.6	0.9
Propensity to consume out of wealth	α_2	0	0.3
Cash to consumption ratio	λ_c	0	0.4
Loans interest rate adjustment	ξ_b	0	0.1
Deposits interest rate adjustment	ξ_m	0	0.1

Table 3: Economically meaningful intervals

Figure 3 shows clearly that the minimisation process for both GDP and the interest rates on bills gave close to a zero difference between observed and predicted values. As mentioned above, the parameters have been estimated so that predicted GDP fits as close as possible to the shocked GDP. Since GDP is the first flow to be fitted, it is normal that the fit is perfect, this will report the shock absorption to other flows. We observe that the parameters determining GDP absorb the shock and divert from their true values.

An other analysis can be made for r_b, the interest rate on bills, figure 4. The interest rate on bills is the last variable to be fitted. Hence, the parameters determining it will absorb all the errors made during the estimation of the previous parameters. If the domain on these determining parameters allow it, the parameters will be calibrated so that the predicted value fits perfectly with the observed value. And indeed we observe large variation in λ_c the desired share of nominal consumption held as cash.

Figure 5 shows that the difference between predicted (blue) and observed (red) interest rates on bonds and government expenditures are larger. These two predicted variable are absorbing the shock applied to GDP. Indeed, the parameters calibrated for these flows, have to cope with the fact that the parameters calibrated for GDP have absorbed the shock and thus have a value different from their true one. In this case, giving the domain constraint for α_1 and α_2, it is not possible to fit perfectly the predicted flows with the observed ones. We nonetheless observe that both variable follow roughly the trend observed in the data. We also observe that both calibrate propensities to consume are similar to the values observed. This shows that even with a shock and differences between already predicted parameters and their observed value, the methodology computes reasonable estimations.

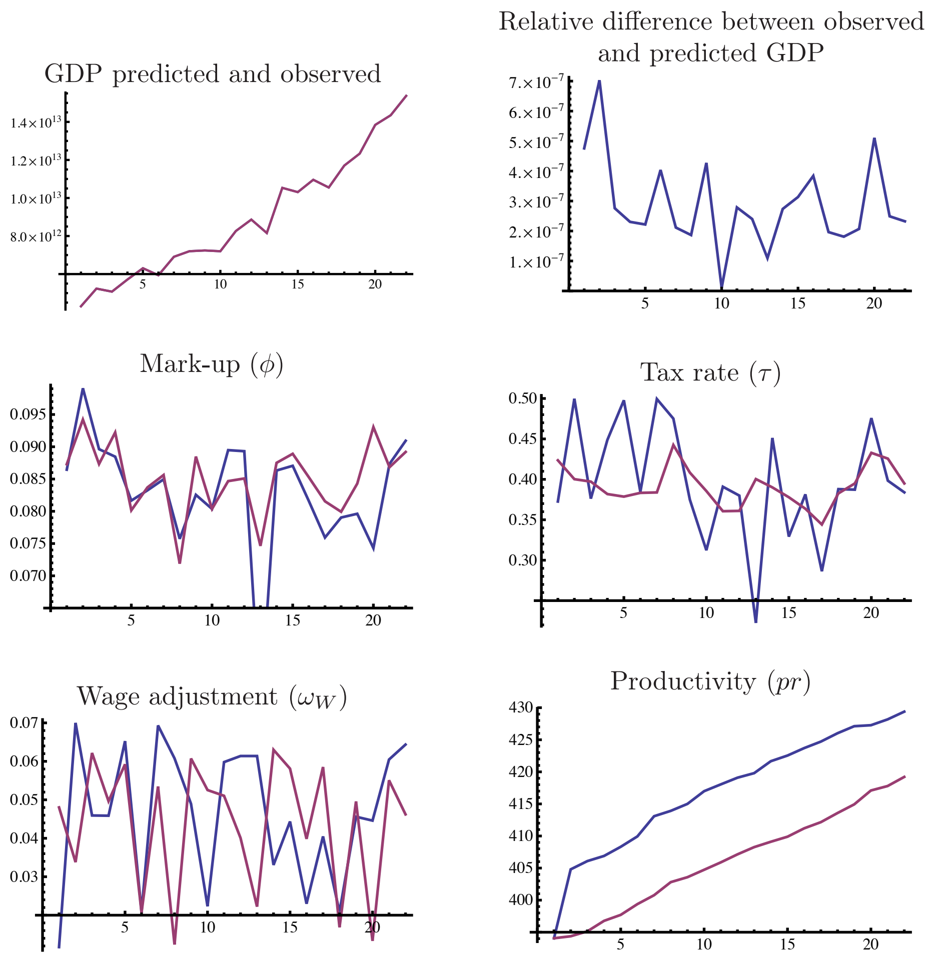

Figure 3: The difference between observed and predicted GDP is almost zero (less than $7 \times 10^{-7}\%$), even with the random shock. We can see that the calibrated parameters (blue) determining GDP absorb the shock and deviate from their generated data (red). The mark-up, which has a more restricted domain follows the trend and is relatively close to its true value. Both the tax rate and the wage adjustment parameters are fairly different from their true values. The productivity parameter follows the same trend but on a higher level.

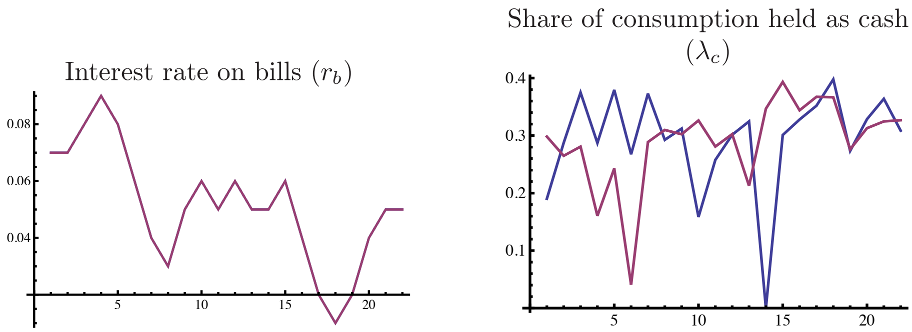

Figure 4: The difference between observed and predicted interest rate on bills (r_b) is almost zero (less than $5 \times 10^{-5}\%$), even with the random shock. This is due to the fact that the domain of λ_c allows the parameter to absorb totally the shock and let the predicted interest rate to fit perfectly the observed data.

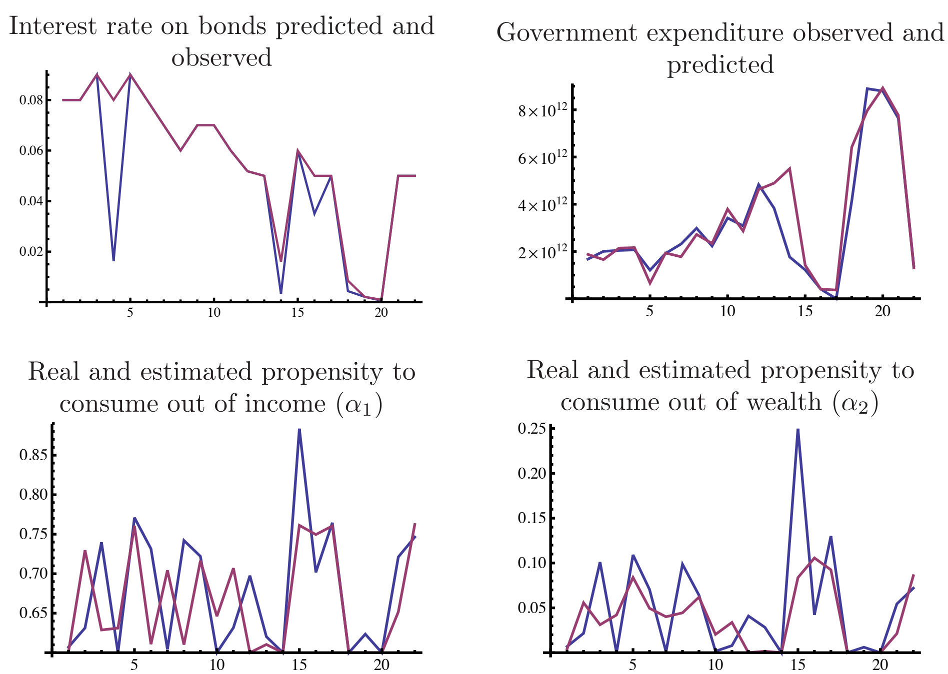

Figure 5: Both the interest rate on bonds and government expenditure variables are absorbing the shock applied to GDP. However, the trends are respected.

5 Conclusion and further work

The goal of this paper is to provide a parsimonious method for estimating stock flow consistent macroeconomic models. We developed an algorithm to solve these large, complicated models, and applied this algorithm to a canonical model of intermediate complexity, the INSOUT model.

The results show clearly that the esimation procuedure works well, producing reliable results from real world data where the parameter values are known.

The objective of this work is to aid in the practical estimation of large models of this type, using a transparent estimation method. It is hoped that other researchers will apply this method to their own stock flow models.

Further work will concentrate on applying this method to a stock flow consistent model of the Irish economy.

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A Appendix

A.1 Variable names

Symbol	**Variable name**
y	Real GDP
N	Employment
WB	Wage bill
UC	Unit cost
s^e	Expected sales
in^T	Targeted inventories
σ^T	Targeted sales to inventories ratio
rr_l	Real interest rates on loans
in^e	Expected inventories
p	Price
$NHUC$	Normal historic unit costs
s	Real sales
S	Nominal sales
in	Real inventories
IN	Nominal inventories
L	Loans
F_f	Firms' profits
π	Inflation
YD_r	Nominal disposable income
CG	Capital gain on long-term bonds
YD_{hs}	Haig-Simons nominal disposable income
F	Total profits
V	Nominal wealth
V_{nc}	Nominal wealth, net of cash
yd_r	Real disposable income
yd_{hs}	Haig-Simons real disposable income
v	Real wealth
c	Real consumption
yd_r^e	Expected real disposable income
C	Nominal consumption
YD_r^e	Expected nominal disposable income
V^e	Expected nominal wealth
H_h^e	Expected households' cash holdings
V_{nc}^e	Expected wealth, net of cash
M	Deposits holdings
Bh	Households' bills holdings
BL	Households' bonds holdings
rr_m	Real interest rate on deposits
rr_b	Real interest rate on bills
rr_{bL}	Real interest rate on bonds
H_h	Households' cash holdings

Symbol	Variable name
T	Taxes
g	Real government spending
$PSBR$	Government deficit
G	Nominal government spending
B	Bills supply
p_{bL}	Price of bonds
r_{bL}	Interest rate on bonds
B_{cb}	Central Bank's bills holdings
H	Central Bank's cash supply
r_b	Interest rate on bills
F_{cb}	Central Bank's profits
H_b	Banks' cash holdings
B_b	Banks' bills holdings
r_m	Nominal interest rate on deposits
r_l	Nominal interest rate on loans
F_b	Banks' profits
Ω^T	Targeted real wages
W	Nominal wages
Y	Nominal GDP

A.2 Parameters name

Symbol	Parameters name
σ_0	Inventories to sales ratio's autonomous parameter
σ_1	Inventories to sales ratio's interest rate slope
γ	Expected inventories adaptation parameter
τ	Tax rate
ϕ	Mark-up
α_1	Average propensity to consume out of income
α_2	Average propensity to consume out of wealth
λ_c	Desired consumption's share hold as cash
λ_{ij}	Portfolio choice parameters, $i \in \{2, 3, 4\},\ j \in \{0, 2, 3, 4, 5\}$
ρ_2	Bank's reserve holding as cash
ξ_m	Deposit interest rate adjustment parameter
ξ_b	Deposit interest rate adjustment with respect to bills' interest rate change
ξ_l	Loans interest rate adjustment parameter
Ω_0	Real wage target autonomous parameter
Ω_1	Real wage target productivity slope
Ω_2	Real wage target employment slope
Ω_3	Nominal wage adjustment parameter

A.3 Equations of the model

$$y = s^e + (in^e - in_{-1}) \tag{20}$$
$$N = \frac{y}{pr} \tag{21}$$
$$WB = W.N \tag{22}$$
$$UC = \frac{WB}{y} \tag{23}$$
$$s^e = s_{-1} \tag{24}$$
$$in^T = \sigma^T s^e \tag{25}$$
$$\sigma^T = \sigma_0 - \sigma_1 r_l \tag{26}$$
$$rr_l = \frac{1 + r_l}{1 + \pi} - 1 \tag{27}$$
$$in^e = in_{-1} + \gamma(in^T - in_{-1}) \tag{28}$$
$$p = (1 + \tau)(1 + \phi)NHUC \tag{29}$$
$$NHUC = (1 - \sigma^T)UC + \sigma^T(1 + r_l)UC_{-1} \tag{30}$$
$$s = c + g \tag{31}$$
$$S = s.p \tag{32}$$
$$in = in_{-1} + y - s \tag{33}$$
$$IN = in.UC \tag{34}$$
$$L = IN \tag{35}$$
$$F_f = S - T - WB + \Delta IN - r_l IN_{-1} \tag{36}$$
$$\pi = \frac{p - p_{-1}}{p_{-1}} \tag{37}$$
$$YD_r = F + WB + r_{m-1}M_{-1} + r_{b-1}B_{h-1} + BL_{-1} \tag{38}$$
$$CG = \Delta p_{bL}.BL_{-1} \tag{39}$$
$$YD_{hs} = YD_r + CG \tag{40}$$
$$F = F_f + F_b \tag{41}$$
$$V = V_{-1} + YD_{hs} - C \tag{42}$$
$$V_{nc} = V - H_h \tag{43}$$
$$yd_r = \frac{YD_r}{p} - \pi\frac{V_{-1}}{p} \tag{44}$$
$$yd_{hs} = \frac{YD_r}{p} - \pi\frac{V_{-1}}{p} + \frac{\Delta p_{bL}.BL_{-1}}{p} \tag{45}$$
$$v = \frac{V}{p} \tag{46}$$

$$c = \alpha_1.yd_r^e + \alpha_2.v_{-1} \tag{47}$$
$$yd_r^e = yd_{r-1} \tag{48}$$
$$C = c.p \tag{49}$$
$$YD_r^e = p.yd_r^e + \pi\frac{V_{-1}}{p} \tag{50}$$
$$V^e = V_{-1} + (YD_r^e - C) \tag{51}$$
$$H_h^e = \lambda_c C \tag{52}$$
$$V_{nc}^e = V^e - H_h^e \tag{53}$$
$$M = V_{nc}^e(\lambda_{20} + \lambda_{22}rr_m + \lambda_{23}rr_b + \lambda_{24}rr_{bL} + \lambda_{25}\frac{YD_r^e}{V_{nc}^e}) \tag{54}$$
$$Bh = V_{nc}^e(\lambda_{30} + \lambda_{32}rr_m + \lambda_{33}rr_b + \lambda_{34}rr_{bL} + \lambda_{35}\frac{YD_r^e}{V_{nc}^e}) \tag{55}$$
$$p_{bL}.BL = V_{nc}^e(\lambda_{40} + \lambda_{42}rr_m + \lambda_{43}rr_b + \lambda_{44}rr_{bL} + \lambda_{45}\frac{YD_r^e}{V_{nc}^e}) \tag{56}$$
$$rr_m = \frac{1+r_m}{1+\pi} - 1 \tag{57}$$
$$rr_b = \frac{1+r_b}{1+\pi} - 1 \tag{58}$$
$$rr_{bL} = \frac{1+r_{bL}}{1+\pi} - 1 \tag{59}$$
$$H_h = V - M - B_h = p_{bL}BL \tag{60}$$
$$T = \frac{\tau}{1+\tau}S \tag{61}$$
$$g = \frac{G}{p} \tag{62}$$
$$PSBR = G + r_{b-1}B_{-1} + BL_{-1} - (T + F_{cb}) \tag{63}$$
$$B = B_{-1} + PSBR - p_{bL}\Delta BL \tag{64}$$
$$p_{bL} = \frac{1}{r_{bL}} \tag{65}$$
$$F_{cb} = r_{n-1}B_{cb-1} \tag{66}$$
$$Hb = \rho_2 M \tag{67}$$
$$r_m = r_{m-1}(1+\xi_m) + \Delta r_b \xi_b \tag{68}$$
$$r_l = \Delta r_b + r_{l-1}(1+\xi_l) \tag{69}$$
$$F_b = r_{l-1}IN_{-1} + r_{b-1}B_{b-1} - r_{m-1}M_{-1} \tag{70}$$
$$\omega^T = \Omega_0 + \Omega_1 pr + \Omega_2\frac{N}{N_{fe}} \tag{71}$$
$$W = W_{-1}\left(1 + \Omega_3\left(\omega^T - \frac{W_{-1}}{p_{-1}}\right)\right) \tag{72}$$
$$Y = ps + UC(in - in_{-1}) \tag{73}$$

A.4 Parameters constraints

$$\lambda_{20} + \lambda_{30} + \lambda_{40} = 1 \tag{74}$$
$$\lambda_{22} + \lambda_{32} + \lambda_{42} = 0 \tag{75}$$
$$\lambda_{23} + \lambda_{33} + \lambda_{43} = 0 \tag{76}$$
$$\lambda_{24} + \lambda_{34} + \lambda_{44} = 0 \tag{77}$$
$$\lambda_{25} + \lambda_{35} + \lambda_{45} = 0 \tag{78}$$
$$\lambda_{23} = \lambda_{32} \tag{79}$$
$$\lambda_{24} = \lambda_{42} \tag{80}$$
$$\lambda_{34} = \lambda_{43} \tag{81}$$

A.5 Transaction matrix

		Households	Firms		Govt.	Central Bank		Banks		Σ
			Current	Capital		Current	Capital	Current	Capital	
Consumption		$-C$	$+C$							0
Government expenditures			$+G$		$-G$					0
Δ in the value of inventories			$+IN$	$-IN$						0
Sales tax			$-T$		$+T$					0
Wages		$+WB$	$-WB$							0
Entrepreneurial profits		$+F_f$	$-F_f$							0
Bank profits		$+F_b$						$-F_b$		0
Central bank profits					$+F_{cb}$	$-F_{cb}$				0
Interest on	loans		$-r_{l-1}.L_{-1}$					$+r_{l-1}.L_{-1}$		0
	deposits	$+r_{m-1}.M2_{-1}$						$-r_{m-1}.M2_{-1}$		0
	bills	$+r_{b-1}.B_{h-1}$			$-r_{b-1}.B_{-1}$	$+r_{b-1}.B_{cb-1}$		$+r_{b-1}.B_{b-1}$		0
	bonds	$+BL_{h-1}$			$-BL_{-1}$					0
Change in the stocks of	loans			$+\Delta L$					$-\Delta L$	0
	cash	$-\Delta H_h$					$+\Delta H$		$-\Delta H_b$	0
	deposits	$-\Delta M2_h$							$+\Delta M2$	0
	bills	$-\Delta B_h$			$+\Delta B$		$-\Delta B_{cb}$		$-\Delta B_b$	0
	bonds	$-\Delta BL_h.p_{bL}$			$+\Delta BL.p_{bL}$					0
Σ		0	0	0	0	0	0	0	0	0

Table 4: Transaction Matrix of INSOUT model.