

The Cold War Hot House for Modeling Strategies at the Carnegie Institute of Technology

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ABSTRACT

US Military needs during the Cold War induced a mathematical modeling of rational allocation and control processes while simultaneously binding that rationality with computational reality. Modeling strategies to map the optimal to the operational ensued and eventually became a driving force in the development of macroeconomic dynamics. Key features of macroeconomics that originated in US military-funded research on applied mathematics in the 1950s and 1960s included recursive optimization taking uncertainty into account, agents modeled as collections of decision rules, and the certainty equivalent and equilibrium modeling strategies. The modeling strategies that opened the door to economists' agile underpinning of macroeconomics with microfoundations had their own foundation in empirical microeconomics. In the 1950's the United States Air Force Project SCOOP (Scientific Computation of Optimum Programs) and the Office of Naval Research awarded contracts to establish and maintain a research center for the Planning and Control of Industrial Operations at

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the Graduate School of Industrial Administration of the Carnegie Institute of Technology in Pittsburgh. Faced with client demands for computable decision rules that would minimize costs in the face of uncertain demand, the Carnegie team reverse engineered effective computational protocols to derive optimal forms and properties. They also developed procedures to narrow the modeling choices in dynamic programming. The formal analysis of the statistical and mathematical properties of bridges connecting the rational with the computable formed a modeling self-consciousness and channeled a new disciplinary focus on modeling strategies. The rational expectations theorists built on the modeling strategies that had been fabricated for planning in their attempt to bridge aggregative economics with the optimizing behavior of the individual and the clearing capacities of markets. The state-transition models that in the 1970s gave macroeconomists a framework for recursive optimization with uncertainty had similarly transformed control engineering in the 1960s. In both disciplines a “modeling-strategy turn” narrowed the primary analytical focus to the properties of mathematical models with less emphasis on the phenomena being modeled.

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In a 1972 Smithsonian Computer Oral History interview, Philip Wolfe told a story about one of his first days at the Pentagon in 1951 employed with the Air Force Planning Research Division's Project for the Scientific Computation of Optimum Programs: "I was wandering around trying to find out what the people in this group were doing. I walked into the office of a man whose name I think was Hortby [?] and asked him what he was doing. And he looked up from his desk and said, 'I'm making a detailed model of the American economy. Please excuse me now. I'm terribly busy.'" The economists and mathematicians at Project SCOOP were constructing input/output tables of the entire US economy in order to plan for a rapid, optimized mobilization in the event of a war with the Soviet Union. The irony was not lost on one particular Secretary of Defense, that it was the Union of Soviet Socialist Republics, not the United States, that wore the ideological mantle of central planning. There is another irony, less obvious but worth exploring, that the sculpted decision framework that emerged from the US military planning initiated by Project SCOOP eventually transformed macroeconomic theory with models that extolled the efficiency of markets and purported to demonstrate the harm that could be inflicted by governments that tried to plan and intervene. One phrase goes a long way to explain this unlikely connection of centralized planning tools and anti-planning theory: modeling strategy.

"New classical economics means a modeling strategy," so asserted Robert Townsend (quoted in Klamer 1983, 81). For Thomas Sargent, the notion of rational expectations was synonymous with the "equilibrium modeling strategy." Sargent was explicit about its effectiveness for bearing fruit through its restrictions on modeling choices,

Research in rational expectations and dynamic macroeconomics has a momentum of its own. That momentum stems from the logical structure of rational expectations as a modeling strategy, the questions that it invites researchers to face, and the standards that it imposes for acceptable answers to those questions. (Sargent 1982, 382)

How did economics reach a point where *modeling strategy* became a key driving force to the development of dynamic macroeconomic theories of how the world works? The modeling strategies that opened the door to economists' agile underpinning of macroeconomics with microfoundations had their own foundation in empirical, prescriptive microeconomics. The mindset that generated the equilibrium modeling strategy of rational expectations developed from a military imperative for contracted applied mathematicians and mathematical social scientists to map the optimal into the operational. This imperative arose in the context of a Cold War between the two superpowers the USA and the Union of Soviet Socialist Republics.² In a secret document chronicling control and management in the US Air Force Air Material Command during the year of 1953, a USAF historian described the competing demands that shaped the mathematical programming of logistics operations,

In the half-light of a cold war, the people could not visualize a possible atomic slugging match. It is always difficult to throw oneself enthusiastically into a warm-up practice for a game which one hopes will never be played. As a result, there was developing within Congressional psychology and the national mind the conviction that taxes and budget deficits must take priority over defense matters....

The increasing resistance to military spending was forcing the Air Force to place greater emphasis upon managerial controls of its operations. ...

It was evident that under conditions of less than full mobilization, the Congress and the people were insisting upon well-planned, orderly, and economical programs. (Miller, 1954, 1-3)

The need to simultaneously reduce military budgets and guarantee readiness in event of the rapid escalation of a Cold War into a hot, nuclear war led to an emphasis on technological design and operational planning through mathematical modeling. As the economist Merrill Flood, wrote in a letter dated December 18, 1945 to Warren Weaver, the head of the World War

² Many historians date the Cold War between the USA and USSR from 1947 to December 1991 when the Union of Soviet Socialist Republics dissolved into more independent nation states.

II Applied Mathematics Panel, “the methods of the analyst should surely be applied to ‘between war’ problems of statesmanship and military planning, as one way to simplify our task before it has begun.”³

In this between-wars setting, the US military underwrote the design of mathematical protocols for optimal allocation processes and optimal control of air-borne weapons and vehicles. The protocols had to model real processes, acknowledge uncertainty, and yield dynamic analysis. The military usually required numerically-specified rule-based solutions to problems. Faced with those conditions, applied mathematicians could not be content with formulating theorems that proved existence and uniqueness of a solution without indicating what the nature of the solution was. The researchers working for a client, had to specify the objective and solve the problem, preferably with swiftly-converging approximate solutions rather than time-consuming exact solutions. Similarly, the solutions had to be in the operational form of a simple rule of action for use by non-mathematicians. The requirements for allocative decision-making, effective computation and communicable decision rules induced a mathematical construction of rationality while simultaneously binding that rationality with computational reality.

The formal analysis of the statistical and mathematical properties of bridges connecting the rational with the computable and operational formed a modeling self-consciousness, and channeled a new methodological focus on modeling strategies - a “procedural rationality” as Herbert Simon would come to call it. In their attempt to bridge aggregative economics with the

³ Flood’s letter to Weaver is in the folder on AMP Notes No. 27, in the box on Weaver theory on Air Warfare in RG 227 in the National Archives. The Applied Mathematics Panel operated from 1942 to 1946 under the auspices of the US National Defense Research Committee (NDRC), which President Roosevelt had set up to do research into mechanisms of warfare. The Applied Mathematics Panel (AMP) included pure mathematicians, statisticians, and economists.

optimizing behavior of the individual and the clearing capacities of markets, the rational expectations theorists built on specific modeling frameworks that had been used in military and production planning problems as well as the general appreciation of the fruitfulness of a regime shaped by the disciplined practice of a modeling strategy.⁴

The legacies of the wartime protocols for macroeconomic theory include

- recursive, dynamic optimization incorporating uncertainty
- the individual agent modeled as a collection of decision rules
- the metaphor of the “benevolent social planner” supplanting that of the invisible hand
- a macroeconomic monoculture of modeling strategizing wherein the avowed primary focus for analytical attention is on the mathematical properties of the models rather than traits of the phenomena being modeled
- a formalized communal sharing of models among modelers and agents in their models
- the drafting of a narrow history of postwar macroeconomics that portrayed the demise of Keynesian economics as the case of an inferior technical revolution defined by equation restrictions inevitably being superseded by a superior technical revolution also defined by equation restrictions

Production Planning at the Carnegie Institute of Technology

During the early years of the Cold War, the US Air Force and Navy financed research programs at the Carnegie business school that demarcated management science by situating it in a decision-theoretic framework. That context generated new models and solution strategies for planning in the face of uncertainty. The research on modeling strategies for production planning

⁴ The focus of this essay is on the background to the modeling-strategy turn in post-war normative microeconomics and macroeconomics. Mary Morgan’s 2012 book, *The World in the Model: How Economists Work and Think*, chronicles a broader history of modeling as a method of enquiry and a reasoning style in economics in the nineteenth and twentieth centuries.

and control also initiated dual academic pursuit curves into modeling economic man: bounded rationality and rational expectations. Both approaches yielded Nobel prizes in economics; indeed the unique Carnegie Institute culture ultimately nurtured the research of seven Nobel Prize winners in economics (see Table 1).

The foundation for several of these prize-winning abstractions was “applications-driven theory.” The starting points were teams of engineers, economists, and statisticians developing models and solution protocols that would give their military/industrial clients optimal rules of action for fulfilling economic criteria through accessible computation. The interdisciplinary teams at the Graduate School of Industrial Administration Business (GSIA) sought scientific ways of formulating accurate expectations of future demand and deciding on the best monthly production rate, workforce size, and inventory level. The mathematics had to yield a rule that a middle manager could specify with only a few hours’ use of a desk calculator and that a non-mathematically trained operative could recalculate in five minutes once a month. The monthly recalculations of the rule had to yield numerical values for production rates and workforce size, which if implemented would lead to smoother production and lower costs than informal managerial judgments. The limitations on computing capacity and data drove the theoretical developments.

Reflecting back on his GSIA experience and explaining his naming of the concept of “applications-driven theory”, the economist William Cooper described the Carnegie approach:

First solve the problem being addressed-up to and including its successful application. Then be sure to turn to the literature in order to perfect, extend, and generalize these formulations. Finally, write up the thus "generalized" formulations for publication and also report results from the application as, inter alia, a way to move into still further applications, and so on.

There is another aspect of this strategy that needs to be observed. All this research was conducted in the form of team efforts. In each case, at least one of the team was a member of the company and was responsible (at least in part) for improving the processes or the operations being addressed. ... These approaches to research, which involved testing in actual use, followed by extension and refinement for publication, became a major component of our research strategy.” (Cooper 2002, 38)⁵

The theoretical developments that emerged from the military-funded applications at the Carnegie business school went even further than Cooper described. They took at least three forms: new generalizations of models and solution protocols for management science; theory designed to structure the mathematical proofs of the optimal properties of the new modeling strategies for decision making; and theory developed to model how humans or organizational structures actually made decisions.

A five million dollar endowment from William Larimer Mellon in 1948 had founded the Graduate School in Industrial Administration at the Carnegie Institute of Technology (see GSAI Milestones in Table 2). George Leland Bach, the Dean of the GSIA and former head of the CIT Economics department, Herbert A. Simon, professor of administration and chairman of the GSIA Department of Industrial Management, and the economist William Cooper designed the first curriculum. From its first academic term in 1949-1950, the GSIA cultivated its niche with an emphasis on an analytical, problem-solving approach to business decision-making. The program conscientiously combined economics, engineering, and a social science of organizational behavior to achieve the goal of “sensitizing of students’ minds to new, and even radical-appearing, ideas and ways of doing things in the industrial administration field” (Bach 1951, 2). The aim was to replace traditional crude rules-of-thumb with decision rules based on mathematics and statistical investigation. Herbert Simon elaborated on the origins of the GSIA in

⁵ Cooper and Leigh McAlister first articulated the concept of applications-drive-theory in their 1999 essay on “Can research be basic and applied?”

a 1975 interview, “We felt like we were going to have the first business school that had academic respectability, scientific respectability, and we didn’t think it needed to run like dead-headed old-fashioned business schools” (McCorduck, 1975, 1). The GSIA marketed their interdisciplinary analytical decision-making approach in contrast to the case-studies approach of what they perceived as a key rival, the business school at Harvard University. As early as 1954, the GSIA was asserting that they had “pioneered in the development of new mathematical and statistical techniques for the more effective handling of business and industrial problems” (Bach 1954, 8). By 1959, Herbert Simon, the acting Dean, would write in the GSIA annual report of 1958-59, “This particular blend of fundamental social science and mathematics with business applications; of economics with behavioral science and organization theory is more and more being identified by colleagues at other institutions with the ‘Carnegie group’” (Simon 1959a, 4).⁶ The GSIA team helped reorient what had been called “operations research,” but what they preferred to now call “management science”. In 1953, Cooper, became the founding national president of The Institute of Management Science (TIMS), and other Carnegie faculty including Abraham Charnes, and Herbert Simon, served as national officers in the first years of TIMS and on the editorial board of its new professional journal, *Management Science*.⁷

⁶ In his study of Herbert Simon, Hunter Heyck (Crowther-Heyck 2005) explains Simon’s instrumental role in establishing the interdisciplinary social science team approach to problem-solving that led to the GSIA becoming a key model for the establishment of new business schools across the USA in the 1950s and 1960s. Heyck argues that in program development, grant proposals, and faculty hiring and firing, Simon consciously structured teams of faculty and graduate students for developing behavioral models of control and choice founded on the combination of project-based practice, basic research, and normative reform, all mediated through the language of mathematics.

⁷ Operations research had emerged from World War II quantitative evaluations of specific military operations or alternative weapons systems. Phillip Morse and other World War II military operation researchers founded the Operations Research Society of America (ORSA) in 1952. The Carnegie team suggested that the distinction between operations research and management science was analogous to that of chemical engineering and chemistry respectively. ORSA and TIMS began sponsoring joint meetings in 1974 and formally merged in the Institute of Operations Research and Management Science (INFORMS) in 1995. The early history of two organizations is documented in Thomas 2015, Gass and Assad 2005, Salvesen 1997, and Flood 1956.

Engineering and quantitative control courses were a strong component of the graduate curriculum. In the early years of operation, the GSIA insisted on an undergraduate engineering and/or science background for perspective students for their Masters of Science in Industrial Administration. There was ongoing debate among the staff as to whether they were “pushing the use of mathematics too far” (Bach 1952, 1-2). The analytical, mathematical approach to decision-making, however, was encouraged and underwritten by two large US military grants. The research funds provided by a military client expecting computable, quantifiable decision rules motivated GSIA professors to develop the following theorems and modeling strategies highlighted in this history:

- Abraham Charnes’s degeneracy theorem for making optimizing linear programming problems operational
- Herbert Simon’s reverse engineering of Laplace transforms to derive the optimal criterion function, which turned out to be a quadratic function cost function
- Simon’s certainty equivalence theorem for demonstrating that in the case of minimizing a quadratic cost function the expected value, even in the absence of knowledge of the probability distribution, is equivalent to a certain value
- Charles Holt and Peter Winter’s forecasting protocol of exponentially weighted moving averages (EWMA) incorporating trends and seasonal components
- John Muth’s reverse engineering of the ad hoc EWMA to derive optimal properties of a permanent component (random walk) and transitory component (white noise)
- Muth’s rational expectations strategy of equating of expected values with equilibrium values that in conjunction with the certainty equivalence theorem led to the equating of equilibrium values with certain values

Intra-firm Planning for USAF Project Scoop

In 1949, the Air Force's Project SCOOP (Scientific Computation of Optimal Programs) and the Bureau of the Budget's, Division of Statistical Standards awarded the GSIA a three-year grant for research on "Intra-Firm Planning and Control" under the direction of William Cooper. The US Air Force (USAF) had set up Project SCOOP in June 1947 (named as such in October 1948) to develop high-speed computation of its own planning process.⁸ George Dantzig, Project SCOOP's chief mathematician, tackled this mission with his articulation of the linear programming model and the simplex computational method for solving linear programming problems.⁹

Limited computing capacity in the late 1940s prevented SCOOP researchers from fully implementing Dantzig's mathematical solutions for Air Force programming. So the economists and mathematicians in Project SCOOP broke the linear programming problems down into triangular models for computationally achievable, but sub-optimal, decisions on monthly

⁸ The history of Project SCOOP from 1947 to its disbandment in 1953 is covered in the chapter on "The Bounded Rationality of Cold War Operations Research" in Erickson, Klein, et. al. 2013. In the context of Project SCOOP's work in the late 1940s and early 1950s the term "programming" did not refer to computer coding as it does nowadays. As Marshall Wood and George Dantzig (1949,193-194) explained, "Programming, or program planning, may be defined as the construction of a schedule of actions by means of which an economy, organization, or other complex of activities may move from one defined state to another, or from a defined state toward some specifically defined objective. Such a schedule implies, and should explicitly prescribe, the resources and the goods and services utilized, consumed or produced in the accomplishment of the programmed actions."

⁹ Linear programming is a prescriptive approach to resource allocation. It tells the user the optimal mix of activities needed to accomplish a task – the least-cost network for shipping from factories to distributors, the best food basket for maximizing nutrients subject to a budget constraint or the cheapest blend of alloys subject to specifications. The linear programming model consists of a linear objective function minimizing losses, or maximizing gains, and a set of linear inequalities that constrain the allocation of scarce resources. Dantzig developed the simplex algorithm for mathematically framing and computing the solutions for the system of linear inequalities such that costs for an activity, subject to resource constraints, would be minimized. With the simplex method investigators descend mathematically the edges of a convex polyhedron, vertex by vertex, until they reach a minimum. Dantzig's computationally efficient algorithm for solving calculation-for-optimal allocation problems had a major impact on the economics and management science professions. In his August 5, 1948 briefing to the Air Staff, Dantzig asserted that "one ranking mathematical economist at a recent conference at Rand confessed to me that it had remained for Air Force technicians working on the Air Force programming problems to solve one of the most fundamental problems of economics."(U.S. Air Force Planning Research Division 1948, 10). Till D ppe and E. Roy Weintraub (2014) discuss the intersection of linear programming and economic science at the Cowles Commission's 1949 conference on Activity Analysis and the shared value of rigor and enthusiasm for convexity analysis engendered at the conference.

requirements for bombs, fuel, ammunition, spare parts, and personnel for specific war plans or maintaining war-ready peacetime levels. The “triangular procedure” mimicked the hierarchical structure of decision making and communication of orders in the Air Force, and the Project SCOOP team implemented it on different scales. On the grandest scale of preparing for World War III, the SCOOP approach started with a war plan based on strategic guidance from the top echelon of the Department of Defense. The next step was to model the inputs of Air Force items and the outputs of Air Force activities that could achieve this war plan. The last step was the input-output modeling of the peacetime economy for the entire USA that would be producing the items necessary for mobilization in the next war. The SCOOP team built their wartime Air Force activities model and their peacetime economy models on Wassily Leontief’s inter-industry input/output analysis and each of the models was further broken down into triangular forms to ensure computability.¹⁰ Project SCOOP and Bureau of the Budget awarded a contract to the Carnegie business school for intra-firm analysis that would complement the data collection and activity planning in the government’s inter-industry peacetime model. The task of GSIA was to study existing data reporting and production planning systems and design improved ones that could more effectively respond to changes in external demand arising from wartime plans:

The object of the project on intra-firm planning would be to analyze processes for making the production decisions and other decisions within the firm responsive to changing “external” information - e.g., markets and customers’ orders, prices, changes in financing opportunities, and so forth. ...

As adequate knowledge is gained of the response of the planning mechanisms of individual firms to external changes, a second area of theoretical work will develop, dealing with **the linkage of the individual firm to the economy, and to external (e.g. wartime) planning mechanisms**. (Cooper, Rosenblatt, Simon 1950 2, 5, emphasis is mine)

¹⁰ For connections between Dantzig’s linear programming and Leontief’s input/output analysis of inter- industry data see Dantzig 1963, 16-18, Kohli 2001, Klein 2001b, 128-133.

Under the contract between Project SCOOP and the GSIA, the subject areas of economics, administrative decision-making, and accounting had to be related through mathematics, statistics, and logic. All the GSIA staff had to have training in one or more of these methods, every research team had to include someone with engineering experience. Mathematics was the stated common language of the project participants. The grant enabled the GSIA in 1952 to set up a joint curriculum between the Departments of Mathematics and Economics leading to the first Ph.D. degree in Mathematical Economics in the USA. The basic research emphasis on abstract mathematical economics was also evident in the Air Force-funding generation of at least four articles in *Econometrica* (Charnes 1952, Charnes, Cooper, Mellon 1952, Charnes, Cooper, Mellon 1954, Simon 1952b).¹¹

To aid the war-related military mission of Project SCOOP, the Carnegie group established the criteria that the companies selected for their intra-firm study “should be either of such dominant importance or of such a representative character that empirical findings will throw direct light on total industry behavior” (Cooper 1952, 3). For example, armed with data from the Philadelphia Gulf Oil Refinery, Cooper and Charnes worked with an engineer from the refinery staff, Bob Mellon, to model linear programming problems for chemical processes.¹²

Although willing to finance basic research, the military client expected the Carnegie group to “spell out the operational significance of these findings for non-mathematicians.” (Cooper 1952, 5). Similarly the Carnegie developers of the intra-firm mathematical protocols had to develop models and solution algorithms compatible with scarce computational resources. Cooper (1952,

¹¹ Philip Mirowski (2002, 452-472) discusses Simon’s Air Force-funded work with the Cowles Commission that was occurring at the same time as the Air Force Project SCOOP grant to the GSIA. Mie Augier (2000) focuses on the influence of Ford Foundation patronage on Simon’s research on behavioral science.

¹² The optimizing approaches by Charnes and Cooper on the one hand and Simon and Holt on the other is also discussed in Erickson, Klein, et. al. 2013.

4) also assured the Air Force that, “the stage of fundamental research should not be regarded as complete until the tools have been developed and simplified to a point where they can be understood and manipulated by practicing administrators and makers of policy [at the level of the firm].” They had to get down to what they referred to as the “handbook stage.”

The strong influence of this client orientation is evident in the developments stemming from the first project on blending aviation gasoline in order to achieve optimal octane levels. Cooper, an economist, was unsuccessful in solving the problem through Tjalling Koopmans’ activity analysis approach that he had been introduced to in a Cowles Commission conference in 1949 (see Koopmans 1951). Cooper (2002) recollected that he approached Charnes from the mathematics department, who upon reading the Cowles conference proceedings on activity analysis focused on Dantzig’s more practical use of linear inequalities. With their first empirical application of linear programming to the problem of blending aviation fuel, the Carnegie researchers were faced with the fact that “unresolved computations could not be expected to proceed safely without the services of a trained professional mathematician at each stage... the work could not be routinized to the degree necessary for large-scale continuing application to practical administrative problems” (Cooper 1952, 4). To solve that problem, Charnes (1952) developed a general means of handling degeneracy and unbounded solutions. In combination with Dantzig’s simplex algorithm, this technique reduced the computation to a simple mechanical routine. Charnes, Cooper, and Mellon first presented their mathematical protocol at the 1951 Project SCOOP symposium on linear programming and published their results in *Econometrica* a year later. The petrochemical industry subsequently became the exemplary user of linear programming and simplex algorithms partly because optimization for the capital-

intensive chemical processing did not require the computational capacity that other industries or the complex Air Force models required.

This first Air Force project at the Carnegie business school illustrates the development of two types of theory: 1) the development of a solution strategy compatible with existing computing capacity that enabled the generalization of linear programming models for industrial applications; 2) the mathematical proof that the new strategy yielded optimal operational solutions amenable to effective computation. This in turn generated considerable corporate interest in linear programming, and Charnes and Cooper eventually consulted with over 100 companies and government agencies. This spurred their development of chance-constrained programming and goal programming. It also meshed well with the GSIA commitment to Project SCOOP and the Bureau of the Budget to train future workers in government and private industry in linear programming and optimization. After training their own GSIA staff and circulating mimeographed lecture notes to colleagues at other institutions, Charnes and Cooper developed a graduate course for the Carnegie business school; the textbooks based on their lectures on linear programming at the GSIA (Charnes, Cooper, and Henderson 1953) and on its industrial applications (Charnes, Cooper, 1961) became standard management science texts.

In contrast with the linear programming approach of their colleagues on the USAF project, Herbert Simon and engineer-turned economist Charles Holt appropriated mathematical approaches to modeling information flows in the feedback loops in servomechanisms to study and design systems in firms that could respond to changing external conditions.¹³ In a schema of

¹³ Simon (1950, 3) provided an accessible definition of the feedback-based, error-actuated servomechanisms: “a device that controls the operation of a machine, perceives the amount of error in its control and acts to correct the error... a servomechanism is a machine that has a purpose (for example, keeping the house at a comfortable temperature), and which strives to accomplish that purpose (by turning on the furnace when it is cool, and turning it off when it is warm).” The control engineering literature that Simon and Holt used relied heavily on work in World

optimizing philosophies, Holt (1951) and Simon perceived this focus on a model of an adaptive mechanism as a polar opposite, to that of linear programming. For Simon at least, designing an adaptive system that could respond in a rapid, stabilizing manner to uncertain changes was often a better way of confronting uncertainty than explicit prediction followed by optimization. For example, in their comments in a 1955 paper on firm planning by Modigliani and Sauerlender, Cooper and Simon asserted,

Servomechanical analogies have an additional appeal in suggesting a useful distinction between what might be called “rational” behavior and “adaptive” behavior.” The traditional model of economic man has been that of a being who continually strives to attain optimal positions. The behavior of such a creature might be termed “rational”....

The adaptive system seeks to assure adjustment to the future *whatever it may be*, rather than optimal adjustment to a future that is predicted and described in terms of probability distributions. (Cooper, Simon 1955, 357-358)

In what the GSIA team called the “Air Force Project,” Simon and Holt hoped that the modeling framework of a servomechanism could serve as a template for firms to design adaptive systems for responding to changing external information from the market. They applied a

War II on lead computing gun sights and predictors and on a common mathematical language of control mechanisms that Warren Weaver encouraged in the Applied Mathematics Panel (see LeRoy A. MacColl’s 1945 study of the *Fundamental Theory of Servomechanisms*, Judy Klein’s, unpublished, *Protocols of War and the Mathematical Invasion of Policy Space, 1940-1960*, and Peter Galison’s 1994 essay on “The Ontology of the Enemy: Norbert Wiener and the Cybernetic Vision,”).

Holt and Simon’s interest in applying servomechanism analysis to planning and control in firms extended past the Project SCOOP contracts to the ONR projects on “Planning and Control of Industrial Operations” at the GSIA and the Air Force-funded Cowles Commission project on “Decision Making under Uncertainty.” They explained how servo analysis could be useful in the seemingly very different environment of planning:

It happens that there is a class of mechanical and electrical automatic control mechanisms usually referred to as servo mechanisms that can be said to “make decisions” by reacting rather than by forecasting and planning, and there is a body of theory available for predicting their behavior. In studying the inventory and production problem it may prove useful to examine the servomechanism analogy for the points of view that may be suggested and the analytical tools that may be borrowed. (Holt and Simon 1954, p. 75)

They argued that the useful insights from the servomechanism analogy included an acknowledgement of the problems of instability inherent in closed-loop feedback systems and the importance of measuring responses of the overall system to external variables. Because costs were incurred by both forecasting and planning performances, they hypothesized that “a simple decision reaction decision rule that relates input variables directly to action decisions may be easier to study and to apply than one that separates the forecasting and planning steps” (Simon and Holt, 1954, 76). In his biography of Herbert Simon, Hunter Heyck (Heyck-Crowther 2005) discusses Simon’s interest in servomechanisms and the recurring theme of adaptation in his research pursuits.

servomechanism model to derive decision rules for scheduling production for the White Motor Company truck assembly plant that would minimize manufacturing and inventory costs.

Engineers working with servomechanisms in World War II had used the operational calculus of Laplace transforms to make computation easier. In that spirit of effective confrontation of limited computing capacity, Simon (1952a, 2) asked under what conditions can the optimal production paths be represented “by linear differential equations with constant coefficients amenable to the use of Laplace transforms?” His reverse engineering of a successful operational approach in order to determine the properties of the optimizing function associated with it led Simon to the conclusion that “in order for Laplace transformation methods to be strictly applicable, the cost function to be minimized must have the quadratic form” (1952a, 4)

Simon’s procedure of reverse engineering a successful, computationally-practical form to reveal a compatible optimal form, as well as the specific use of the quadratic cost function for easing computation, would carry over into the GSIA planning project for the US Navy. In 1953, the Secretary of Defense, Charles E. Wilson, began implementing President Eisenhower’s significant cuts to the defense budget and eliminating any programs that smacked of planning the US economy. Project SCOOP, with its large research planning budget and its goal to use an inter-industry input/output model of the US economy to plan for wartime mobilization plans got the axe in 1953.¹⁴ The GSIA transferred their intra-firm planning and control studies still in progress for the US Air Force to the project on producer logistics for the Office of Naval Research (ONR).¹⁵ It was in the ONR research project that the polar optimizing philosophies of

¹⁴ With Wilson at the helm of the Department of Defense, the Air Force’s Planning Research Division, which had sheltered Project SCOOP, quickly changed to its name to the “Computation Division.” Wilson’s anathema to planning and its effect on the disbandment of Project SCOOP is discussed in the chapter on “The Bounded Rationality of Cold War Operations Research” in Erickson, Klein, et. al. 2013.

¹⁵ In May 1948, Mina Rees, the director of the Mathematical Science Branch of the Office of Naval Research, arranged for George Dantzig and his Project SCOOP colleagues to make a presentation on linear programming and

the adaptive closed feedback loop (characteristic of servomechanisms) and mathematical programming were brought much closer together with the GSIA appropriation of Richard Bellman's dynamic programming protocol for adaptive control in a multi-stage decision process.

Rules of Action for the Office of Naval Research

The ONR contract for a GSIA research center and major project on the "Planning and Control of Industrial Operations" began in the fall of 1952, led by the engineer/economist Charles Holt. Cooper, Charnes, Holt, and Simon were on the ONR research team as well as macroeconomist Franco Modigliani, Carl Lemke, Robert Culbertson. These were joined later by Peter Winters and John Muth. Their project began with interviews of managers from 15 companies to identify problems in production planning (Holt 2002, 96). The group selected the Springdale paint manufacturing plant of the Pittsburgh Plate Glass Company (now PPG Industries), for the focus of their development of mathematical protocols for deriving decision rules for specifying production rates, inventory levels, and work force strength. In one of the GSIA's first ONR research memoranda, Simon described the conditions facing production managers at the Springdale plant (called the Viscous Manufacturing Company to preserve company anonymity):

The system under study consists of a "viscous" factory, including a factory warehouse, and a group of twenty-seven geographically dispersed district warehouses and stores supplied from the factory. ...

A wide variety of items is manufactured. There are some 500 types of viscous, and each type is sold in a number of can sizes. In all, about 1,500 items are regularly manufactured and stocked, and other items are made on special order. A relatively small number of items make up a large part of the total gallonage, but maintenance of a full line of products is an important objective.

scientific planning to all the Rear Admirals in the Navy. The result of that encounter was the Navy's establishment of the Logistics Research Project (see Rees 1972, 22 and U.S. Air Force Planning Research Division 1948, 9). The scale of the ONR funding for mathematical research was impressive- over \$29 million in 1949 alone - and one grateful recipient, Alexander Mood, referred to Rees as "the angel of mathematics at the Office of Naval Research" (Shell-Gellasch 2002, 883-884).

Most of the items are subject to wide seasonal fluctuations in sales. (Simon 1953, 1)

Decision rules for determining optimal inventory levels were important to the paint factory and ultimately to Naval logistics. For the company, inventories helped to smooth production and employment requirements and avoid stock run-outs and the need to manufacture uneconomically small batches. There were, however, costs to holding inventories. Other mathematical social scientists under contract with the Office of Naval Research, including Kenneth Arrow, Theodore Harris, and Jacob Marschak (1951) and Aryeh Dvoretzky, J. Kiefer, and Jacob Wolfowitz (1952a, 1952b) had used iterative schemes to provide existence proofs for optimal inventory control solutions. The Paint Factory Project team made reference to these, but had a different goal of developing decision rules that could be numerically specified with reasonable computational resources. By August 1957 the Carnegie team had produced over 54 ONR research memoranda. The Navy research team's most important output, however, was the influential text, *Planning Production, Inventories and Work Force* by Holt, Modigliani, Muth, and Simon (published in 1960 and abbreviated in the operations research community as HMMS).

For the Navy project, the Carnegie team took a dynamic programming approach that yielded optimal linear decision rules for an organization.¹⁶ The Carnegie group's programming contributions to management science, economic theory, and modeling strategies included

¹⁶Richard Bellman, a mathematician at the Air Force Project at the RAND Corporation, had developed the rule-based dynamic programming approach to recursive optimization in the early 1950s (see for example Bellman 1952, 1954, 1957, Klein 2007a, and the chapter on "Bellman Strikes Gold in Policy Space with Dynamic Programming" in Klein's unpublished *Protocols of War and the Mathematical Invasion of Policy Space*). Working on Air Force projects, Bellman was struck by the utility of a possible solution of a mathematical problem being a rule of action and he developed a way for solving dynamic programming models by numerical approximation in what he called "policy space". Solving for the optimal policy was a dual approach to the original primal problem of solving for the value of minimum costs in an allocation problem. Bellman simplified the problem of finding the optimal policy for a multistage problem by assuming that whatever the decision taken in the initial stage, the remaining decisions must be an optimal policy built on that first decision. Dynamic programming was essentially an algorithm for determining the optimal allocation of resources over time in contrast with linear programming's optimal allocation of resources across activities.

modeling costs with a quadratic criterion function, forecasting expected demand in the presence of trends and seasonal components with exponentially weighted moving averages (EWMA), deriving linear decision rules that firms could easily quantify and use, and developing rigorous tests for the performance of their rules and forecasting methods. To complement this management science, the team developed mathematical proofs and explorations of the optimal properties of the certainty equivalent theorem related to a quadratic criterion function and of the EWMA forecasting model. There was, however, a third tier of theory that grew out of the Navy project on production planning: formal reflections on how individuals and organizations make rational decisions with limits on their information and computational resources.

Approximating Costs with Quadratic Functions

With the stimuli of the USAF Project SCOOP and the ONR funding, the primary initial focus of GSIA research was on the theory of the firm. “Not the firm of the economic textbooks,” as Simon later explained, “but the firm as it appeared to managers who had to make decisions in the presence of uncertainty.” In describing the core team of Holt, Modigliani, Muth, and Simon that worked for years on the “Paint Factory Project” for the ONR, Simon (2000b, 6) claimed that “uncertainty became our common bond.” The researchers had brought with them a prior interest in uncertainty, and as a team they focused on future demand for a type of paint and its heavy draw on the information-processing capacity required for making optimal production decisions. That common bond of uncertainty yielded not just a thorough generalization of the forecasting method based on exponentially weighted moving averages, but also a new modeling strategy and theorem such that the more easily accessible point estimate of expected value of future paint sales could stand in for the entire probability distribution.

A formal statement of the objective of a problem was a key component of the decision-theoretic framework constructed in programming and planning projects for the US military in the late 1940s and early 1950s. The explicit objective function or criterion function formally stated the maximizing or minimizing goal that would guide the decision-maker. In dynamic programming problems it was usually something along the lines that total costs will be at their minimum and that minimum is a function of an optimal policy combination of variables under one's control to deal with the "state" or "environmental" variables.

For a production manager at the Pittsburgh Plate Glass paint factory, a minimization of production and inventory costs was the objective and the variables under the manager's control were the inventory and production levels achieved within the upcoming month. The environmental variables included the initial values at the beginning of the month of the level of inventories and the existing rate of production. Future demand for paint was an environmental variable that was not only outside of production manager's control but also uncertain. The ubiquitous presence of uncertainty in planning for future demand forced those turning toward mathematical solutions to draw on statistics and probability distributions. Mathematicians had demonstrated that in the case of uncertainty the probability distribution of the uncertain variable could stand in for the certain value, but grasping the entire distribution placed high demands on data and computational resources. Previous dynamic programming allocation studies for the military had gotten around this by assuming time series were stationary and joint distributions were the product of independent distributions for each time period. In factory settings, the data needed to fully construct probability distributions of future demand were hard to come by, and the presence of trends and seasonal influences and other factors of the business environment rendered the assumptions of stationary time series and independent distributions unrealistic.

Confronted with the need to incorporate uncertain demand and the need to minimize the use of scarce computational resources, Holt and Simon (1954) hypothesized that if the criterion function was in a quadratic form, then the dynamic programming problem could be solved with merely the use of a point estimate of expected value of demand for each future time period. The expected value could, for example, be estimated by a moving average of past values. In addition to eliminating the need to specifying the entire probability distribution, including the variance and higher moments, a quadratic cost function meant that the optimal decision rules could be solved with a matrix that was computationally easy to invert. In an ONR research memorandum on techniques for the solution of dynamic programming problems, Simon, Holt, and Modigliani explained the reasons for restricting the criterion function to a quadratic form:

Under the limitations of present knowledge, we are faced with a choice between attempting to deal with the real situation in all its complexity in the face of almost insuperable mathematical and computational problems; or simplifying the problem to make its solution and the practical application of that solution exceedingly easy. We believe that the particular form of the cost function proposed here buys a great deal in the way of simplicity and compatibility at a relatively small cost in accuracy....

The strategy adopted here is to incorporate these simplifications in the criterion function, and to do so in such a way that the resulting problem can be solved exactly. (Simon, Holt, and Modigliani 1955, 4, 6-7)

Armed with a quadratic approximation to the behavior of production and inventory costs, Holt, Modigliani, and Simon (1955a, 1955b) were able to calculate generalized linear decision rules for establishing the optimal production rate, size of the workforce and level of inventory of final goods that would result each month in minimum costs. At the Pittsburgh paint company, the Carnegie team measured the time it took a production manager to initially specify the model and to recalculate it each month and they compared the actual performance of the factory over six years with the hypothetical performance that would have been realized had the factory been

using the quadratic criterion function and the optimal decision rule. They concluded that following the prescribed mathematical protocol would have reduced costs by 8.5% annually (Holt, Modigliani, and Simon 1955b, 2). The team boasted that once the decision problem was formalized in the way they suggested, the numerical constants “could be computed with a desk calculator in three man-hours,” and once the parameters of decision rule were established, computation of monthly decisions “required only a five-minute calculation” (Holt, Modigliani, and Simon 1955a, 2). In justification of the easily computed quadratic cost function, the Carnegie team argued that the U-shaped cost curve with the minimum being the optimal point was a good fit for the economic theory of costs in the short-run and for what managers expected. Inventory above the optimal level incurred unnecessary storage costs and inventory below that level meant higher costs associated with increased number of unforeseen machine setups or penalty costs associated with runouts, back orders, and delayed shipments.

Holt, Modigliani, and Simon asserted that the quadratic criterion function leading to a linear decision rule could be applied with little drain on computational resources to many factory settings and a wide-range of decision problems. In addition to this generalization for management science and dynamic programming, Simon proved mathematically that when the criterion function of a dynamic programming problem was reasonably estimated by a second-order approximation in the form of a sum of quadratic and linear terms, the “planning problem for the case of uncertainty can be reduced to the problem for the case of certainty, simply by replacing, in the computation of the optimum first period action, the ‘certain’ future values of variables by their unconditional expectations.” (Simon 1956a, 74).¹⁷ In other words, the point

¹⁷ In his 2009 essay, “A Feasible and Objective Concept of Optimal Monetary Policy: The Quadratic Loss Function in the Postwar Period,” Pedro Duarte examines the use of the quadratic criterion function in the management science work at the Carnegie Institute of Technology, Henri Theil’s influential rationale for using the function in his 1964 book on *Optimal Decision Rules for Government and Industry*, and the function’s important role in modeling the

estimate of the expected value of a variable at the end of the first period (for example sales at the end of the coming month) was the equivalent to the certain value at the end of the month. Simon called it his “certainty-equivalent method” and his “certainty-equivalence theorem.”

Forecasting with Exponentially Weighted Moving Averages

Once Simon had demonstrated that the expected value of future sales was the equivalent of the certain value, the next challenge for the paint factory team was to decide how to estimate the expected value for each of hundreds of types of paint while minimizing computational resources. The team tried simple moving averages of past sales, but they were introduced to a more computationally efficient and accurate forecasting protocols by way of a November 1956 presentation at a ORSA annual meeting by Robert G. Brown on “Exponential smoothing for predicting demand.”¹⁸ Brown specialized in inventory control at the management consulting

central bank’s optimal behavior in monetary economics beginning with William Poole’s 1969 use of the function for determining optimal choice of monetary policy instrument. The computational feasibility of the quadratic loss function also played an essential role in modeling adaptive control processes that characterized US military-funded research for maintaining optimal trajectories in space vehicles (see Bellman 1957, Beckwith 1959, Kalman 1960, Kalman and Bucy 1961). The assumption of a linear dynamical system and a quadratic loss function reduced difficult dimensionality in dynamic programming problems. It also enabled Rudolf Kalman and Richard Bucy to use the solution of the Riccati differential equation for estimating and deriving the optimal properties of their linear filter that minimized expected-squared error (cost) in the estimation of a signal generated by a linear stochastic differential equation on a Hilbert space. Indeed, in control theory the Kalman-Bucy filter algorithm is synonymous with linear quadratic estimation (LQE).

¹⁸ During World War II the US Applied Mathematics Panel developed the mathematical model of the information flow in the analog computer of gun sights in turrets of B17 and B24 bombers. The mechanical ball-cage integrators and gyroscopic computers essentially exponentially smoothed the inputs from the marksmen’s angular rotation of a gun as he tried to keep the enemy fighter plane in their sight. The gunner’s angular rotation served as a proxy for the relative velocity of the fighter plane and the computer fed back the smoothed output by disturbing the gun sight thus affecting the marksmen’s rotation. The gunner’s arm movements, the analog computer, and the disturbed sight thus constituted an error-actuated servomechanism.

During the mid-1950s the equation worked its way into the operations researchers’ and economists’ communities via control engineers who had encountered it in their work on analog computers. For example, Bill Phillips, an engineer, passed the equation onto the economist Milton Friedman who passed it on to Philip Cagan. The two economists introduced their new model of adaptive expectations to the macroeconomic theories on the consumption function and hyperinflation respectively (Friedman 1957; Cagan 1956). Holt’s inspiration came from the operations research community rather than from Friedman or Cagan. In the late 1940s, Brown, a Yale-educated mechanical engineer had used the equation in his work on the analog computers on depth control charges in anti-submarine warfare for the Navy’s Operations Evaluation Group. The World War II and Cold War origins of the EWMA model and subsequent diffusion are discussed in the chapters on “Bomber turrets, Ball-cage Integrators, and the Applied Mathematics Panel” and “Exponentially Weighted Moving Averages in Operations Research” in Klein’s unpublished *Protocols of War and the Mathematical Invasion of Policy Space*.

firm of Arthur D. Little. Starting in 1956, Brown recommended that clients forecast future sales by computing a weighted average of past sales for a product in such a way that the most recent observations had the greatest weight. This adaptive approach to forecasting smoothed the random variation of demand. In early 1957, Holt went to visit Brown at the Arthur D. Little headquarters in Cambridge Massachusetts. In April of that year Holt came out with a variation on the protocol that smoothed not only the random variation but also the trend and seasonal components. Holt explained his forecasting method in an ONR memorandum entitled “Forecasting Seasonals and Trends by Exponentially Weighted Moving Averages.” Holt’s memorandum was soon widely circulated in the operations research community. Reflecting back on the widespread application of the exponential smoothing protocol, Holt described its virtues: “it was easy to program, fast to compute, required minimal data storage, put declining weight on old data, used simple initial conditions, had robust parameters, was automatically adaptive, model formulations were easy, and the math was tractable” (Holt 2004, 124).

The HMMS team incorporated forecasting by exponentially weighted moving averages (EWMA) into the production planning protocols with this equation stating that predicted sales during this month were based on the prediction of sales in the previous month plus a fraction of the difference between actual and predicted sales in the previous month:¹⁹

$$\bar{S}_t = \bar{S}_{t-1} + w_e (S_{t-1} - \bar{S}_{t-1})$$

\bar{S}_t = expected sales rate in period t
 S_{t-1} = actual sales in previous period

¹⁹ This was the equation given in Holt et. al. 1960, 260, but there were variants of it in Holt’s 1957 ONR memoranda. If you substitute in the equation for \bar{S}_{t-1} and then for \bar{S}_{t-2} and so forth, for each substitution backward you must multiply the coefficient $(1 - w_e)$ by itself an additional time. Given that w_e is a positive number less than one, the weights decline in a geometric progression, thus the name exponentially weighted moving averages. One can also see from this equation that the forecaster needs only to store information from the single previous period because that captures information from all past stages.

w_e = weighting or smoothing parameter with a value between 0 and 1

Peter Winters (1960), a GSIA graduate student, wrote a computer program for an IBM 650 to test the accuracy of EWMA forecasts against other methods. The EWMA forecasting protocol proved more accurate than other methods used at that time and lowered the draw on computational resources. The team gained further insights on applications and limitations of the EWMA forecasting protocol with John Muth's 1960 study of "Optimal Properties of Exponentially Weighted Forecasts."²⁰ Muth's goal was to "characterize the time-series for which the exponentially-weighted forecast is optimal" (Muth 1960, 301). He took up the modeling strategy that Simon, his senior colleague, had pursued in his first derivation of a quadratic cost function. As Simon had done in reverse engineering linear differential equations amenable to Laplace transform to get to the minimizing variational equation, Muth worked backwards from the relatively accurate, easily computable ad hoc EWMA forecasting model that had caught fire in the business, operations research, and macroeconomic theory worlds to see what structural model of random processes ended in the solution taking the form of the exponentially weighted moving averages being the conditional expected value. Muth demonstrated that the model that yielded the EWMA as an optimal solution was composed of two random processes: a random walk whose influence persisted through all subsequent time periods plus white noise whose influence did not persist past a single time period. Thus, there was a permanent component and a

²⁰ Muth's educational trajectory exemplified the GSIA selection process and mission. He entered the Carnegie graduate program with an undergraduate degree in industrial engineering from Washington University, earned a Masters of Science in Industrial Administration in 1954, and was appointed an instructor in the GSIA in 1955. Muth completed the requirements for a Ph.D. in Industrial Administration, with Simon signing off on his doctoral thesis, in 1962 (see Muth 1962).

transitory component and the best forecast for the next period was also the best forecast for all future periods, because the “best” forecast only estimated the permanent component.²¹

During the last four decades of the twentieth century, the Holt-Winters method of using an EWMA model with exponentially smoothed trends and seasonal components became one of the most widely used business forecasting formula.²² The team’s quadratic approximation of the costs curve associated with inventory, back order and machine setup costs, their equation for exponential smoothing of past sales to get the expected value of a future month’s sale, and their decision rules for determining the month’s aggregate production rate and the size of the workforce for that month are reproduced in Figure 1. The Pittsburgh paint factory application, however, drove Carnegie-group theory beyond just decision science generalization or novel modeling strategies and mathematical proofs of optimal properties to new conceptualizations of how humans made decisions.

Bounded Rationality

In a 1958 article in the *Harvard Business Review*, the Carnegie team claimed that their mathematical approach to decision making was “more rational than judgment based on experience and informed hunch” because it was more precise and consistent and it translated

²¹ In applying EWMA to model aggregate consumer behavior in an economy, Milton Friedman (1957) had argued that consumption depended on the values of permanent and transitory income. Superficially Muth’s structural approach appeared to lend support to Friedman’s permanent income hypothesis, but Muth made clear that his structural model was not the same as Friedman’s “vague” decomposition of permanent and transitory components of income (Muth 1960, 304). Friedman (1957, 21) saw the permanent component as “analogous to the ‘expected’ value of a probability distribution” and the transitory component as including chance occurrences as well as fluctuations resulting from the business cycle. Friedman admitted that there was “leeway” in the precise, practical meaning of the “permanent” component (Friedman 1957, 225-226). The work of Friedman and Cagan is discussed in the chapter on “Adaptive Expectations in Macroeconomic Theory” in Klein’s unpublished *Protocols of War and the Mathematical Invasion of Policy Space*.

²² The influence of Holt’s 1957 memorandum on the time series analysis of George Box and Gwilym Jenkins (1970) as well as the influence of Muth’s 1960 study on developments in structural time series analysis with recursive residuals is discussed in the chapter on “Exponentially Weighted Moving Averages in Operations Research” in Klein’s unpublished *Protocols of War and the Mathematical Invasion of Policy Space*.

“from a language that permits the implicit to a language that compels the explicit.” (Anshen, Holt, Modigliani, Muth, and Simon 1958, 53). This compulsion of the explicit, however, had laid bare the constructed rationality’s great demands on data gathering and computing resources. In the first research memorandum to the Navy in which they introduced their quadratic criterion function, Simon and Holt (1954, 15) summarized the “substantial progress” they had achieved in making rational decisions about production and inventory, but they warned that their limited computing and information capacities were sending the theory in novel directions: “Our theory development should be guided by the limits of our data. If our estimates of cost functions are going to be, at best, extremely rough, then one property of a good decision rule is that it not be highly sensitive to changes in the constants.” (Simon and Holt, 1954, 15). Practicing what they preached, the Carnegie team eventually settled on a quadratic cost function and linear decision rule for which overestimating costs by 100% or underestimating by 50% would subsequently incur costs only 11% higher than would have been the case with the correct estimates (Anshen et. al. 1958, 55).

The data and computational limitations on rational decision making revealed by the compulsion of the explicit led to Simon’s articulation of *bounded rationality*. Simon and Holt declared in one of their first Navy research memoranda justifying the use of a quadratic criterion function for their dynamic programming protocol,

There should be no impossible gulf between “theory” and “practice”: good theory is theory that works. A theory that tells you that if you knew a man’s utility function, or if you could forecast sales perfectly, you could make an optimal or rational decision gives little comfort—unless the numbers that have to be substituted in the equations are available or can be supplied. A theory of rational decisions is a theory of how to decide, given certain kinds of information and certain computing capacities. For these reasons, we have avoided drawing any sharp line between the “theoretical” and the “rule-of-thumb,” and have tried to emphasize instead how good theories, and the

basic intuitions derivable from them, can be of direct assistance to the operating man in making his production and inventory control decisions. (Simon and Holt 1954, 16-17)

This was a major break from the standard economic approach to rationality that assumed the decision maker had perfect information and unlimited computational capacity. Those assumptions had given economic theorists license to confine rationality and their analytical attention to the decision outcome of maximization without having to examine the process of how to decide. Perfect knowledge and unlimited computing capacity were behind the game theorist's assertion that players maximized expected utility and the microeconomist's assumption that firms maximized profit by choosing the quantity of production that equated marginal revenue to marginal costs.²³ Simon, the model-maker who had to search for approximations to make optimizing protocols conform to limited computational resources of his client, argued that model-makers in the positive, descriptive economics realm should model consumers and entrepreneurs as likewise approximating to make do with limited computational resources.

In a 1953 RAND Corporation paper that Simon had worked on simultaneously with the paint factory project, Simon proposed a model "for the description of rational choice by organisms of limited computational capacity" (Simon 1953, 1). He illustrated the advantage of focusing on the actual decision-making process of a chess player with a comparison of a utility function for someone who had unlimited computational power (the S-shaped curve in Figure 2) with one for someone whose limits on computational capacity made ideal optimization too costly

²³ This was not the first time Simon challenged economists and their discipline. In his study of Herbert A. Simon, Hunter Crowther-Heyck documents the tensions spanning decades between Simon and his Carnegie colleagues in economics. Nor was this the first time that social scientists, or Simon himself, seriously contemplated notions of limited rationality. In their essay on "The Conceptual History of the Emergence of Bounded Rationality" (*History of Political Economy* 37, no. 1 (2005): 27-59), Matthias Klaes and Esther-Mirjam Sent construct a conceptual trajectory of first time occurrences of a family of expressions of limited, approximate, incomplete, and bounded rationality

(the stepped line). If limits on computational capacity constrain the ability to evaluate all alternatives (such as all possible future plays in a chess game) then the player will seek an approximating mechanism to simplify computations. For example, the payoff function could be flat over a wide range of alternatives and the utility function would be characterized by plateaus that map three simplified alternatives such as “clearly won”, “clearly lost” and “clearly drawn”. In the words of Simon (1953, 9), “The player, instead of seeking for a ‘best’ move, needs only to look for a ‘good’ move.”

As Simon and his team had discovered, when confronted with imperfect information and limited computing power in the practice of optimization, operations researchers often had to adapt their prescriptive models by resorting to approximation or adapt their approach by resorting to being satisfied with an aspirational level. In 1955, the year that Holt, Modigliani, and Simon published their “Linear Decision Rule for Production and Employment Scheduling” there was no operational electronic digital computer at the Carnegie Institute and most academic, corporate, and military institutions relied on desk calculators. The Carnegie team’s chief theoretical contribution to management science that year was the demonstration that approximating costs with a quadratic function made industrial applications of dynamic programming possible because such an approximation required only a few man-hours at a desk calculator. They introduced a model and a modeling strategy that overcame limitations on computation capacity. In that same year, Simon published his paper on “A Behavioral Model of Rational Choice” in the *Quarterly Journal of Economics*. In that essay, he argued for “less-global models of rationality” that involved “new considerations –in particular taking into account the simplifications the choosing organism may deliberately introduce into its model of the situation in order to bring the model within the range of its computing capacity.” (Simon 1955, 112, 100).

In his 1957 book on *Models of Man*, Simon named his “principle of bounded rationality.” He argued that consumers and entrepreneurs were “intendedly rational,” but they had to construct simplified models of real situations that were amenable to effective computation. Although the economics actors acted rationally with regard to their model, the construction of the model was limited by capacities and thus related to the psychological properties of “perceiving, thinking, and learning” (Simon 1957, 199). For Simon the key to simplifying the choice process and reducing computational demands was “the replacement of the goal of *maximizing* with the goal of *satisficing*, of finding a course of action that was ‘good enough’” (Simon 1957, 204). This meant accepting that the intendedly rational decision maker, optimizing with respect to a simplified model, behaved in a suboptimal way with respect to the real world. It also meant that the economics discipline would be better served by incorporating more psychology in understanding human behavior and by modeling the process of reaching equilibrium not just the state of equilibrium (Simon 1959b, 263).²⁴

For Simon, bounded rationality was destructive in its critique of models of economic man, but constructive in the potentials for new theories of organizational behavior and digital computer simulations of complex decision processes including adaptive and satisficing behavior. Limited rationality was at the core of Simon’s organization theory: his substitution of a “choosing organism of limited knowledge and ability” for economic man provided the rationale for administration man and the formal study of organizations that had occupied much of his research energy: “This organism’s simplifications of the real world for purposes of choice introduce discrepancies between the simplified model and the reality; and these discrepancies, in turn, serve to explain many of the phenomena of organizational behavior.” (Simon 1955, 114)

²⁴ Simon would later call this theoretical focus on the process of choice, as opposed to the results of rational choice, “procedural rationality”.

Similarly, Simon turned to artificial intelligence and computer simulations of cognitive problem solving to investigate adaptive, satisficing behavior.²⁵ In 1978, Herbert Simon was awarded the Sveriges Riksbank Prizes in Economic Sciences in Memory of Alfred Nobel for his "pioneering research into the decision-making process within economic organizations."

In his Nobel Prize lecture in Stockholm on December 8, 1978, Simon reflected on the conceptual development of bounded rationality by describing the Carnegie team's quadratic cost approximation to illustrate "how model building in normative economics is shaped by computational considerations." (Simon 1979b, 498).²⁶ That same Carnegie team application however, initiated another seemingly very different conceptual development. What John Muth took from the experience of developing modeling strategies for the paint factory was the need to augment the assumption of rationality in modeling expectations.

The Rational Expectations of Entrepreneurs in a Market

The Navy research program on the Planning and Control of Industrial Operations had highlighted the importance of incorporating uncertainty and expectations in models and the inadequacy of neoclassical economic theory, with its focus on static equilibrium, in dealing with this. In 1955, the Carnegie business school hosted a Social Science Research Council conference on "Expectations, Uncertainty, and Business Behavior." Prominent economists, psychologists,

²⁵ Hunter Heyck (Crowther-Heyck 2005) and Ester Mirjam Sent (2000, 2005) have elaborated on the disciplinary span of Simon's work, particularly the shift from an interest in organizational decision making to computer emulation of human cognitive problem solving and the accompanying travel across academic departments at Carnegie Mellon University that included industrial administration, psychology, and political science. In contrast, Mie Augier (2000) emphasizes a singular thread in Simon's professional work over decades. In *Models of a Man: Essays in Memory of Herbert A Simon*, (edited by Augier and March 2004), family, friends and colleagues reflect on Simon's contributions to studies in modeling, artificial intelligence, and organizations. Gerd Gigerenzer and Reinhard Selten's collection of essays on *Bounded Rationality: the Adaptive Toolbox*, examine subsequent interpretations and applications of Simon's challenge to cognitive science, economics, evolutionary biology, and other disciplines.

²⁶ The normative economics that Simon referred to is prescriptive economics as opposed to descriptive economics. It is the application of economic concepts and models to tell a client what ought to be, as opposed to positive economics that describes what is.

and industrial researchers gathered at the Carnegie Institute to discuss theories of expectations, psychological approaches, business practices, and forecasting (see Bowmans 1958). As mentioned earlier, starting in 1957, Charles Holt and Peter Winters began generalizing, for business forecasting, the exponential smoothing model that economists were calling adaptive expectations.

In his 1959 essay on “Theories of Decision-making in Economics and Behavioral Science” Simon called for the modeling of the formation of expectations such that it “incorporates in the theory the process of acquiring knowledge about that environment. In doing so, it forces us to include in our model of economic man some of his properties as a learning, estimating, searching, information-processing organism.” (Simon 1959b, 269) The traditional maximizing model builders had assumed omniscience: the search process was instantaneous, costless and complete. Simon argued there was a learning production process, there was a cost to this information gathering process, and model builders therefore had to pursue an analytical narrative of the economy of the learning process.

Simon did not offer his own model of costs of information-gathering in the formation of expectations, but he did hold up models of the Abraham Wald’s (1947 & 1950) sequential analysis and Jacob Marschak’s (1955) work on the theory of communication in teams as exemplary bridges between economics and psychology. These military-funded studies had applied marginal analysis in their normative treatment of the costs of the observation process. Wald argued that the decision maker should learn from each additional observation and should take into account extra gain and extra cost in deciding in favor of the new gun, in favor of the old gun, or in favor of gathering more information before making a decision on guns. The economist rule book of marginal analysis, however, was written by the master craftsmen of maximizing-

behavior models: “Information, says price theory, should be gathered up to the point where the incremental cost of additional information is equal to the incremental profit that can be earned by having it. Such an approach can lead to propositions about optimal amounts of information gathering activity....” (Simon 1959b, 270).

Simon saw the admission of the necessity for considering the cost of information gathering as challenging the neoclassical assumption of omniscience, but he acknowledged that the examples he was describing of models of how information should be gathered retained “a classical picture of economic man as a maximizer.” (Simon 1959b, 270). For Simon, the existence of an information search process destroyed the assumption of perfect foresight forcing a redefinition of rationality that had to include learning, adaptation, and a modeling of the costs of information-gathering. Contemporary theories on optimal amounts of information-gathering activity, however, had had to resort to the old-style rationality. Simon’s Ph. D. student, John Muth took that as a cue for embedding a super, unbounded rationality within his boundedly-rational theory of the formation of expectations.²⁷

A year after Simon published his essay on theories of decision-making, Muth took Simon’s line of reasoning down a very different path than Simon had envisaged. Muth cited Simon’s 1959 essay on decision-making theories and paraphrased Simon’s notion that the assumption of rationality led to theories inadequate to explain dynamic phenomena. Then Muth countered with the bold hypothesis “based on exactly the opposite point of view: that dynamic economic models do not assume enough rationality” (Muth 1960, 316).

²⁷ Esther-Mirjam Sent (2002) elaborated on the importance of the Carnegie context on Muth’s work, the sequenced, complex interplay of bounded rationality and rational expectations in his professional writing, and the unintended consequences of his microeconomic work appropriated to the realm of macroeconomic theory. Sent used interviews and correspondence with Muth to illustrate his own assessment that macroeconomists misread his seminal paper on expectations and misrepresented his ideas. She documents his subsequent work on the statistics of extremes and on operations management that questioned the appropriateness of the use of normal distributions and quadratic approximations and took Muth more into the realm of bounded rationality.

In his *Econometrica* article on “Rational Expectations and the Theory of Price Movement,” Muth was investigating the notion that fluctuations in prices in an isolated market could result from the mere fact that business expectations are subject to error. He argued that what was missing in this theory of fluctuations was an adequate explanation of how expectations are formed. Muth formalized the hypothesis that entrepreneurial expectations were the same as the predictions of the economic theory of a market.

Even though his research was underwritten by the Navy’s contract for the Planning and Control of Industrial Operations, Muth made it clear that he was trying to explain how business firms form expectations (not how they should be formed), and how that formation of expectations influenced price movements in a market. He went so far as to assert that the naming of the expectations as “rational” was to signal a positive, rather than a normative, economic theory:

In order to explain these phenomena, I should like to suggest that expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory. At the risk of confusing this purely descriptive hypothesis with a pronouncement as to what firms ought to do, we call such expectations “rational.” Muth 1961, 316.

The assertion that “information is scarce, and the economic system generally does not waste it” was central to his modeling strategy. That principle of conserving information meant that the subjective probability distributions of entrepreneurs were the same as the objective probability distributions that arise from predictions of economic theory. Muth voiced a rationale of his assumption of the conservation of information that sounded much like the “efficient market

hypothesis” that would gain ground a few years later and serve as a close traveling companion to the scholarship of rational expectations:²⁸

If the prediction of the theory were substantially better than the expectations of the firms, then there would be opportunities for the “insider” to profit from the knowledge –by inventory speculation if possible, by operating a firm, or by selling a price forecasting service to the firms. The profit opportunities would no longer exist if the aggregate expectation of the firms is the same as the prediction of the theory... if expectations were not rational there would be opportunities for economists to make profits in commodity speculation, running a firm, or selling the information to present owners. Muth 1961, 318, 330

The efficient market for information would ensure that any possible advantage arising from an individual entrepreneur’s successful application of an economist’s economic model would have been seized upon by competitors as soon as they sniffed profits. The aggregate expectations of entrepreneurs for the market price in the future will therefore be no worse than the equilibrium price predicted by economists. Muth’s recognized legacy was in his hypothesis that the average firm’s expectations are “essentially the same as the predictions of the relevant economic theory,” and thus “expected price equals the equilibrium price” (Muth 1961, 315, 318). That assumption turned out to an effective modeling strategy that ensured that the parameters describing entrepreneurs’ (or consumers’) expectations disappeared from the analysis as the expected price was replaced by the equilibrium price.

²⁸ Eugene Fama (1965a and 1965b) and Paul Samuelson (1965) independently developed the mathematical hypothesis of the efficient market: at any moment in time market prices fully reflect all available information. In efficient markets the price is “right,” the change in price is a random variable, and future price changes are unpredictable. That precludes not only investors beating the market over a long period of time without inside information but also asset price bubbles. The dominant narratives of the financial crisis of 2007-2009 that laid out the causes of the housing bubbles in the US and Europe and the policy prescriptions for the consequences of the bursting of the asset price bubbles called into question the assumptions of both the efficient market hypothesis and the market-clearing macroeconomic theory linked to rational expectations.

As a model maker himself, however, Muth, faced with the usual limitations on informational and computational capacities, had to engage in some satisficing behavior in the leap from equating the subjective and objective probability distributions to equating the entrepreneurs' expected price to the economists' theoretically-determined equilibrium price. He had to assume a quadratic profit function to be maximized and to rely on Simon's proof that in such a case expected values are the equivalent of certainty and knowledge of entire probability distributions is unnecessary for solving the mathematical problem. So in proving his case for the assumption of regarding expectations as a rational dynamic model based on maximizing behavior, Muth had to resort to a satisficing short cut. Just as Simon had found it difficult to fully analyze bounded rationality without drawing on a maximizing model of the information search process, Muth found it necessary to rely on a make-do, computationally-conserving strategy to prove the optimizing quality of his modeling strategy.

Although Muth's essay on rational expectations was published in the prestigious *Econometrica* journal in 1961, the tool he had introduced for modeling away expectations lay unused for several years. In 1962, the GSIA awarded Muth his PhD in industrial administration for his thesis on optimal linear decision rules (see Muth 1962). He subsequently focused most of his research on the design, normative economic analysis, and control of production activities in firms (see, for example, Groff and Muth 1972).

Muth had designed the rational expectations model to explain price instability in a micro-level market for a good subject to a production lag - what economists called a cobweb or hog-cycle model. It was via the subsequent work of Carnegie professors Robert Lucas, Edward Prescott, and Thomas Sargent that rational expectations would eventually dominate the realm of

macroeconomic theory and be called into ideological service as proof of the stabilizing quality of markets that persisted as long as a government did not interfere.

The Rational Expectations of Consumers in an Economy

Robert Lucas, who was a professor in the Carnegie Graduate School for Industrial Administration from 1963 to 1974, had not been a member of the research team on industrial control. He was, however, influenced by the normative focus on planning that the ONR and Project SCOOP- funded research had engendered at the business school. In a 1998 interview with Bennett McCallum, Lucas commented on the important influence of the Carnegie group's research strategy and Muth's work in particular on his own study of rational expectation:

Jack [Muth] was the junior author in the Holt, Modigliani, Muth, and Simon monograph *Planning Production, Inventories, and Workforce*. This was a normative study –operations research—that dealt with the way managers should make decisions in light of their expectations of future variables, sales....

The power of thinking of allocative problems normatively, even when one's aim is explaining behavior and not improving it, was one of the main lessons I learned from Carnegie, from Muth and perhaps more from Dave Cass. The atmosphere at Chicago when I was a student was so hostile to any kind of planning that we were not taught to think: How *should* resources be allocated in this situation? How *should* people use the information available to them to form expectations? But these *should* be an economist's first questions. (Lucas quoted in Samuelson and Barnett 2007, 61-62)²⁹

Robert Lucas's first appropriation of Muth's concept of rational expectations appeared in an unpublished Carnegie Institute of Technology research memorandum in 1966. In "Optimal Investment with Rational Expectations," Lucas asked "how will a competitive industry, composed of optimizing firms, respond to shifts in its demand and factor supply functions?"

²⁹ In Kevin Hoover and Warren Young's (2013) panel discussion on rational expectations, Robert Lucas, Michael Lovell, and Dal Mortensen spoke of the direct impact of the social atmosphere and context of the GSIA at the Carnegie Institute of Technology. For example, Mortensen explained, "as a student, I was trained to create models, take models seriously, and apply them. And that was the milieu that I received"(Hoover and Young 2013, 1173).

(Lucas 1966, 1). In the first part of the study Lucas answered that question by assuming static expectations (see also Lucas 1967). In the section on the effects of dynamic expectations, Lucas assumed “that firms’ expectations are rational i.e. correct” (Lucas 1966, 2). He further clarified that by explaining that he was assuming that “the output price expectations of each firm are rational, or that the entire future price pattern, $p(t), t > 0$ is correctly anticipated” (10). In a footnote to that sentence, Lucas explained that the difference between his and Muth’s notion was the assumption of certainty: “John F. Muth defines a rational expectation as one with the same mean value as the true, future price, where both the expected and actual prices are random variables. With certainty, this definition reduces to that used here” (Lucas 1966, 10).

In his 1966 memorandum, Lucas concluded that under the assumption of rational expectations you could specify industry output and price through time, but not an individual firm’s demand and supply functions because the optimal supply response of any one firm and the industry as a whole had to be determined simultaneously. The behavior of the industry was the same with static or rational expectations, but under the latter assumption, “the ‘equations of motion’ of the industry may be interpreted as necessary conditions for the industry-wide maximization of a ‘discounted consumer surplus’ integral” (Lucas 1966, 2).

By the time of his first journal article on rational expectations Lucas had teamed up with Edward Prescott, a GSIA colleague, and further qualified the notion of rational expectations. Lucas and Prescott (1971) made an assumption, similar to that of Muth, of a probability distribution common to both expected and actual output prices in order to model anticipated future demand of firms. They assumed these firms were operating in a competitive industry facing random shifts in the industry demand curve. In the course of determining competitive equilibrium time paths of investments and prices for the industry, Lucas and Prescott paired

Bellman's dynamic programming protocol to Harold Hotelling's (1931) assumption that the competitive industry in essence maximized consumer surplus. As with Lucas's 1966 memorandum, actual and expected prices were simultaneously determined and the firms' optimizing behavior was consistent with industrial equilibrium. Their mathematical narrative demonstrated a stabilizing quality of a free market in which capital stock, buffeted by the changing market forces of demand and supply, settled on an equilibrium level.

Recursive optimization, including Bellman's functional equation theory of dynamic programming and the Kalman Filter enabled Lucas, Sargent and others to construct a "new classical macroeconomics" that embraced the micro foundations of the rational expectations of consumers and producers as well as a notion of dynamic competitive equilibria in the macroeconomic demonstration that money is neutral and anticipated effects of countercyclical government policy often render such intervention impotent or destabilizing. With their appropriated mathematical models of intertemporal decisions processes, they demonstrated that economic actors could rationally anticipate the future effect of many government policy changes and that anticipated monetary expansions would only lead to higher inflation and not to increased output or higher employment (see for example Lucas 1972 and Sargent and Wallace 1975). These mathematically elegant demonstrations in positive macroeconomics thus carried the normative prescription that governments should avoid fine tuning the economy even with the good intention of lowering unemployment, and should instead stick to steady, rule-based guidelines for changing the money supply.

As Lucas and Sargent (e.g. 1981) have acknowledged, Simon's certainty equivalence modeling strategy proved nearly as important to applied dynamic rational expectations theory as Muth's equilibrium modeling strategy. The assumption of a quadratic loss function enabled

rational expectations theorists to separate out the forecasting stage and the non-stochastic optimization problem. Lars Hansen and Sargent voiced a more recent reflection on the advantage of that separation:

By sharply delineating the two steps of (1) optimizing for a given set of expectations and (2) forming expectations optimally, certainty equivalent problems formed a perfect environment for extracting the methodological and econometric lessons of rational expectations. Two of the most important of these were : (a) how rational expectations imposes a set of cross-equation restrictions that link the parameters of an optimal decision rule to laws of motion for variable that influence a decision maker's payoffs (e.g., prices), but that are beyond his control; and (b) how the concept of Granger-causality, based as it is on a prediction-error criterion, can guide the empirical specification of variables that belong on the right side of decision rule because they help predict those influential variables. Environments for which certainty equivalence holds are ones for which it is easiest to compute decision rules analytically. That has facilitated formal analysis as well as numerical computation. (Hansen and Sargent 2004, 2)

The certainty equivalence modeling strategy combined with the equilibrium modeling strategy and the accompanying conclusion of the inherent stability of markets in the absence of intervention soon spread from the microeconomic realm of the firm and industry to the macroeconomic realm of aggregate consumer and investment expenditure. The modeling strategies also enabled the construction of a modeled theoretical edifice connecting these two realms. Other historians of economics have closely examined the construction of this edifice.³⁰

³⁰ Kevin Hoover (1988) offers a thorough methodological treatment of the new classical macroeconomics. Esther-Mirjam Sent (1998) examined the professional trajectory of Thomas Sargent against the backdrop of major changes in economic theory and time series analysis. Drawing on interviews with the participants, Warren Young and William Darity (2001) trace out an early history of developments in adaptive, rational, and implicit expectations in macroeconomic theory. To commemorate the 50th anniversary of the publication of Muth's 1961 paper, Hoover and Young organized "Rational Expectations: Retrospect and Prospect: A Panel Discussion with Michael Lovell, Robert Lucas, Dale Mortensen, Robert Shiller, and Neil Wallace" (2013). The panelists discussed the roles of management science and Simon's certainty equivalence theorem in shaping rational expectations and the revolutionary quality of Muth's equilibrium modeling strategy. Antonella Rancan (2013) examines the evolving similarities and differences in Simon's and Modigliani's approaches to uncertainty and their impact on perceived differences in behavioral economics and rational expectations. Marcel Boumans (2003) analyzes Lucas's change of modeling protocols from dynamic programming to the Kalman filter and his various notions of rationality compared with concepts advanced by Muth, Alan Turing, and Claude Shannon. Roman Frydman and Edmund S. Phelps

Their thorough histories allows for a focus here on Simon's role in forming the model-maker's self-consciousness and his nesting of the HMMS team's certainty equivalence strategy and Muth's equilibrium modeling strategy into his conceptualization of procedural rationality.

The Rationality of the Modeler

In 1956, Simon explained the context for major steps the Carnegie team took in developing modeling strategies:

It is only when one tries to understand the actual mechanisms of decision-making—as distinguished from the classical concern with what a man would do if he shared God's omniscience—that one appreciates the central problem in this kind of rational behavior is to obtain information and to use that information in computations; and that the entire mechanism of decision is molded by information-processing considerations. (Simon 1956b, 2)³¹

The need to minimize or at least operate within information processing constraints had led to other instances of economists working for a client applying quadratic criterion functions in solving problems in normative economics.³² Muth's equilibrium modeling strategy for positive economics was a follow through of Simon's call for conveying the lessons learned about actual decision making in the prescriptive, imperative mood to making models in the descriptive indicative mood; scarce information processing resources for forecasting prices lead entrepreneurs to appropriate the economists' rational model of prices in equilibrium. Muth's

(2013) used the commemoration of the fortieth anniversary of the publication of *Microeconomic Foundations of Employment and Inflation Theory* to untangle research in the late 1960s on microfoundations of macroeconomics from the rational expectations hypothesis it came to be associated with. Microfoundations of macroeconomics were also carefully reconsidered in the University of São Paulo symposium on that subject (ed. Duarte and Lima 2012). Michel De Vroey and Pedro Duarte (2013) have dissected the old and the new neoclassical syntheses and the rise of dynamic stochastic general equilibrium models, which is also the focus of Duarte's 2011 essay on "Recent Developments in Macroeconomics."

³¹ Simon's shift from a decision-making referential framework to an information-processing framework was spurred by this collaboration, beginning in 1952, with experimental psychologists and Allan Newell on simulations of an Air Force information processing center at the RAND Systems Research Laboratory (see Klein 2015).

³² In addition to the HMMS team, Henri Theil (1954) had also resorted to the use of the quadratic function for decision making in national planning for the Netherlands.

strategy was also an antithetical reaction to Simon's suggestion for modeling the limited rationality of producers and consumers. Simon's reaction to Muth's reaction evolved over time, perhaps partly because of his own dual influence on the articulation of the equilibrium modeling strategy. Simon argued that Muth's need to resort to the assumption that any profit function that was being maximized was a quadratic function (or that losses were a quadratic function of the errors of estimates) meant that the "rational" qualifying adjective on expectations was misleading. Because entrepreneurs in Muth's schema could be profit maximizers only under very special circumstances, the adjective conferred an "unwarranted legitimation" to rational expectations models. Simon suggested that "consistent expectations" would have been a more appropriate description of the model (Simon 1978b, 10).

Simon also tried to diminish the significance of rational expectations by arguing that it was a proof of the existence and uniqueness of an optimal fixed-point prediction, but it could not provide a method for making a fixed-point prediction and therefore could not effectively eliminate the problem of uncertainty in economics (see for example Simon 2000). Most significantly, Simon classified rational expectations as an example of "procedural rationality."

In an interview in 1999 in which he was discussing his articulation in the early 1970s conception of procedural rationality, Simon stated,

What I did mostly after- there are a few exceptions to this, but mostly after 1960 was to philosophize about economics. Now, you can call that research if you like. But it was mostly trying to understand really why economics hand [sic] on to these unrealistic models and what were the differences between the way they thought about decision making and the way psychologists did. (Herbert Simon Interview, March 17, 1999).

At a talk at Groningen University in September 1973 and in revisions circulated in 1974 and published in 1976, Simon clarified his emphasis on the problem solving process by making the

distinction between what he called “substantive” and “procedural” rationality. Most economic theory focused on optimal *outcomes*; behavior characteristic of substantive rationality “depends upon the actor in only a single respect – his goals” (Simon 1976a, 130). In contrast, procedural rationality focused on the *reasoning process* of appropriate deliberation including the search for computational efficiency: “a theory of rationality for problems like the travelling-salesman problem is not a theory of best solutions – of substantive rationality – but a theory of efficient computational procedures to find good solutions – a theory of procedural rationality.” (Simon 1976a, 130). Procedural rationality was the modeler’s search for good, or even best, methods of model construction and solution algorithms to derive good solutions to problems; it was “the rationality of a person for whom computation is the scarce resource” (Simon 1978a, 496).³³

In another illustration of procedural rationality, Simon offered the programming model with the quadratic criterion function that his Carnegie team had used for their operations research on production planning: “this did not mean that we thought real-world costs functions were quadratic; it meant that we thought that many cost functions could be reasonably approximated by a quadratic, and that the deviations from the actual function would not lead to seriously non-optimal decisions” (Simon 1976a, 139).³⁴ As with his own certainty equivalence theorem on the quadratic cost function and linear decision rules, Simon declared that Muth’s assumption that

³³ In addition to putting a premium on the procedural rationality of computational efficiency, military-funded research induced economists to consider the most efficient way of implementing the optimal decision rules that emerged from their normative microeconomics. This is documented in Klein’s 2015 conference presentation on “Implementation Rationality: The Nexus of Psychology and Economics at the RAND Logistics Systems Laboratory, 1956-1966.”

³⁴ Simon went on to explain:

Not only did the quadratic function provide good computational efficiency, but it also greatly reduced the data requirements. ...All that was expected of the solution was that the optimal decision in the world of the model be a good decision in the real world. A quadratic function cannot model the more realistic asymmetry of costs about the optimum, but for production planning in the 1950s it was a good-enough approximation. (Simon 1976a, 139)

expectations of firms are distributed about prediction of theory was “only procedural rationality” (Simon 1976b, 4).

Procedural rationality was the “rationality of the modeler” (Simon 1979, 2). But the consumers and producers that economists modeled could also be portrayed as model makers with scarce information and computational resources. The modeled become modelers; in this context it is not difficult to see the leap to the communism of models characteristic of rational expectations. The rational expectations theorists, particularly Sargent, have been explicit about the kernel of modeling strategy at the heart of their revolution. As Sargent explained, “It’s more a technical revolution. ... These are technical issues about staring at models” (Sargent quoted in Klammer 1983, 80).

Modeling-strategy Turns in Macroeconomics & Control Engineering

The state-transition (or state-space) models that in the 1970s gave rational expectations theorists the framework for recursive optimization with uncertainty had similarly transformed control engineering in the 1960s. In both disciplines there was a *modeling-strategy turn*, a re-orientation to a framework emphasizing the mathematical properties of the models themselves with less attention to the properties of the phenomena being modeled. The Kalman filter and other products of optimal control engineering were developed for and applied to empirical phenomena, but the theoretical developments, new concepts, and disciplinary discourse of control engineering and rational expectations were much more likely than pre-1960 to be focused on the mathematical properties of the models. Note the similarity between the following two passages:

“Rational expectations” is a term John Muth coined to refer to a model-building principle It is a property which a mathematically explicit economic model either does or does not possess. One can ask, for example, whether expectations are rational in the Klein-Goldberger model of the United States economy; one cannot ask

whether people in the United States have rational expectations...The limited intelligibility of popular discussions of rational expectations is, I think, entirely due to the understandable desire to evade this central fact. (Lucas, manuscript 1979)

To put it more bluntly, control theory does not deal with the real world, but only with mathematical models of certain aspects of the real world; therefore the tools as well as results of control theory are mathematical. There is a close analogy between this situation and the evolution of probability theory into a strictly mathematical discipline. (Kalman, 1969, 27)

Building on the success of his 1960 design of an algorithmic state-transition filter for maintaining optimal trajectories, Rudolf Kalman and other systems theorists developed a science of model building for designing control systems replete with the articulation of properties such as observability and controllability that all mathematical models of control systems built on optimization and feedback should possess.³⁵ This meta-modeling science, which dwelled more in the realm of mathematical models, with less of a presence in the empirical realm of physical phenomena, came to define control engineering after 1960 and proved reasonably effective: “stability” in one’s mathematical model often ensured that the designed adaptive system was dynamically stable.³⁶

Control engineering for the space race, in contrast to the rational expectations approach to macroeconomics, was a normative *design-for-control* pursuit and many of the parametric values in the mathematical models were gifts of physical laws of motion. Kalman followed the passage quoted above with a concrete example:

³⁵ “A dynamical system is a precise mathematical object; the study of system theory is then largely, although not entirely, a branch of mathematics” (Kalman, Falb, Arbib, 1969, 3-4). Klein 2001a and Klein unpublished document a history of the Kalman filter.

³⁶ There was a similar modeling-strategy turn in linear programming. Alex Orden (1993) recounts how in the late 1960s, linear programming had become two fields. The algorithmic branch aligned with mathematics and computer science in its focus on what Simon would call procedural rationality while the model-building branch remained tied to the traditional roots of operations research and management science and focused on the phenomena being modeled.

Suppose we wish to send a spacecraft to the moon, where it must land on a small prescribed spot. The control theorist would proceed to formulate the problem as follows:

1. To land exactly on a given spot the space craft must be injected into orbit with very high precision and follow a precalculated path without deviation until the landing. This is impossible to accomplish without control because of the difficulty of injecting in orbit, because of random forces acting on the spacecraft between the earth and the moon, etc.

2. A mathematical model for the motion of the spacecraft is provided by the equations of celestial mechanics, which are known to be highly accurate. The physical characteristics of the spacecraft can be assumed to be known, since *we* are building it. The inputs to our system, with which control must be accomplished, will be rockets and jets of various types. The outputs will be measurements of position, velocity, etc., by optical, inertial, or other means.

3. Now we have a precise definition of the plant as a dynamical system. From the properties of the equations of celestial mechanics, it follows that the system is finite dimensional, continuous time, smooth, and (unfortunately) nonconstant and nonlinear....

The spacecraft is to follow a preassigned path until it lands on the moon; the deviations from this given path must be small as possible during the entire flight and especially at landing. This is the *regulator problem*; it amounts to forcing the plant to behave according to a preassigned pattern.

The solution of the regulator problem consists in giving a *control law*, which prescribes the values of the controlling inputs as a function of the measured deviation from the preassigned path. ... We shall be content to obtain the explicit equations describing the regulator, and we shall regard this process as synonymous with “constructing” the regulator.

Now, the all-important point is this: in control theory it is immaterial what physical object is represented by the dynamical system under consideration; what matters is the mathematical structure of the plant....So the problems are strictly mathematical. (Kalman 1969, 27-28)

In contrast, the reorientation in economics to a greater focus on the mathematical properties of models had a mainly descriptive rather than prescriptive design mission. Also macroeconomists had to work with messier laws of motion than those provided by celestial mechanics. Compared with space travel, the modeling-strategy turn in economics has been less

effective in its capacity for helping economists to explain, predict, and control. It has, however, been extremely effective in perpetuating itself, “a momentum of its own” as Sargent (1982, 382) described it.

Legacies of Modeling for a US Military Client

Mathematical protocols of allocative recursive dynamics and control were ripe for Lucas’s, Prescott’s, and Sargent’s picking in the 1970s and 1980’s because the military needs during the Cold War had steered mathematics toward models with a functional equation formulating an economic criterion of maximizing gains or minimizing losses and a policy space for taking the less-traveled algorithmic dual route to an optimal decision rule for thinking at the margin in each stage of a multistage process. Specific modeling strategies, which that had been developed in planning and systems design to effectively map the optimal into the computable, or in the case of reverse engineering, the computable to the optimal, proved useful for bridging optimizing microeconomics and aggregative economics. The real rocket fuel for the momentum, however, came from donning the thinking cap of procedural rationality and staring strategically, and fairly exclusively, at models.

To summarize, the needs of the military client for an applied mathematics that yielded operational rules for actual decision making led to a reorientation in normative economic research at the Carnegie Institute of Technology. The fruit of this research, guided by a procedural rationality, were models and modeling strategies that were characterized by:

- *recursive, dynamic optimization incorporating uncertainty*: As intimidated graduate students have long suspected, macroeconomic dynamics is rocket science; the applied mathematics developed in the Cold War Space race to engineer optimal trajectories has been a big part of the macroeconomist’s tool kit. Lucas, Prescott, Sargent, and other

rational expectation theorist used dynamic programming, as formulated by Bellman and further developed by the HMMS team at the Carnegie Institute, as well as the more specific structure of the Kalman filter to break macroeconomic dynamic problems into multistage processes. At each stage the state is estimated and the transition from that state to the next is mapped. As Lars Ljungqvist and Thomas Sargent (2000, xxiii) described recursive macroeconomic theory, “this enterprise is about discovering a convenient state and constructing a first-order difference equation to describe its motion.” Adhering to the modeling properties of observability and controllability in the systems theory developed by Bellman, Kalman and other control engineers ensures that control is a function of the state, that the state is the minimum information needed to control all future states, and that the state incorporates all information needed to determine the control action that is the key component in the transition to the next state.

- *the individual agent modeled as a collection of decision rules:* Decision rule-based mathematics is at the heart of dynamic programming, optimal control theory, the Carnegie-brand of management science, and recursive macroeconomics. The history of wartime protocols such as sequential analysis and optimal inventory control illustrates how the US military often insisted that the application of mathematics culminate in an operational rule of action (Klein 2006, Klein 2007). Bellman generalized this client proclivity and leveraged it for computation in his construction of a dual framework for solving problems maximizing gain or minimizing loss. The dual approach was that of “approximations in policy space,” which determined the policy rule for specifying the optimal decision in each stage of a process. As part and parcel of their appropriation of the state-transition models for recursive optimization, macroeconomists appropriated the

rule-based mathematics for conceptualizing the representative agent. As Robert Lucas (1986, S401) explained, rational expectation theorists “view or model an individual as a collection of decision rules (rules that dictate the action to be taken in given situations).”

- *the metaphor of the “benevolent social planner” supplanting that of the invisible hand:*

A key to understanding the importance to the US military of Cold War research on optimal allocation processes is to acknowledge that from 1945 to at least until 1962, many at the Pentagon thought the US was between major wars and should be far better prepared for the next world war. The strong pressure to reduce government expenditures from their very high World War II levels, however, put a premium on model-based, research for design, planning, and being ready for swift mobilization of resources. From 1947 to 1953, US military planning for the next war against the centrally-planned Soviet Union entailed input-output modeling of the entire peacetime economy. That was the initial source of research funds for the intra-firm planning at the Carnegie Institute. The optimization characteristic of the normative, welfare economics underwritten by Air Force and Navy funds during the Cold War affected positive, descriptive economics literally and metaphorically. For example, Nancy Stockey, Robert Lucas, and Edward Prescott (1989, 441) wrote:

there is a wide class of situations in which the “invisible hand” ensures that the sets of Pareto-optimal allocations and competitive equilibrium allocations coincide exactly. In these situations we can interpret certain normative models of optimal decision-making (from the point of view of a hypothetical “benevolent social planner”) as positive models of equilibrium outcomes.

- *a communal sharing of models among modelers and agents in their models:*

Normative model-making in the face of scarce information and computational resources encouraged Simon and Muth to ponder how to model the actors that

populated the models of positive economics. The modeled became modelers. As Sargent explained,

All agents inside the model, the econometrician, and God share the same model. The powerful and useful empirical implications of rational expectations—the cross-equation restrictions and the legitimacy of the appeal to a law of large numbers in GMM [Generalized Method of Moments] estimation—derive from that communism of models. (Sargent quoted in Samuelson and Barnett 2007, 312)

- *a macroeconomic monoculture of modeling strategizing* wherein a key focus for analytical attention is on the mathematical properties of the models rather than traits of the phenomena being modeled. The modeling strategies outlined here were not in a vacuum separated from economic theory. New classical macroeconomists linked the rational expectations strategy to their advocacy in the market clearing properties of the economy. The new Keynesians eventually adapted the same strategy to incorporate price frictions and the far-from instantaneous process of adapting expectations to new policy information.³⁷ Since the 1970's there has been, however, a heavier weight of attention and scholarly energy on the mathematical properties of models with a diminished weight on the phenomena being modeled.³⁸
- *an historical narrative of postwar macroeconomics as two technical revolutions* : The rational expectations theorists saw themselves countering one wartime-inspired/funded technical revolution with another. In their presentation, “After

³⁷ See for example, Taylor 1975, Phelps and Taylor 1977, and da Silva's 2015 conference presentation on “The First Keynesian Reactions to Lucas's Macroeconomics of Equilibrium”.

³⁸ Speaking on the state of the discipline from the perspective of the 2008 financial crisis in the US and elsewhere, Ricardo Caballero (2011) remarked, “By some strange herding process, the core of macroeconomics seems to transform things that may have been useful modeling short-cuts into a part of a new and artificial “reality.” And now suddenly everyone uses the same language, which in the next iteration gets confused with, and eventually replaces, reality.”

Keynesian Macroeconomics,” at the 1978 Boston Federal Reserve Conference on
“After the Phillips Curve: Persistence of High Inflation and High Unemployment”

Lucas and Sargent explained that for them:

The Keynesian Revolution was, in the form in which it succeeded in the United States, a revolution in method. This was not Keynes’s intent, nor is it the view of all of his most eminent followers. Yet if one does not view the revolution in this way, it is impossible to account for some of its most important features: the evolution of macroeconomics into a quantitative, scientific discipline, the development of explicit statistical descriptions of economic behavior, the increasing reliance of government officials on technical economic expertise, and the introduction of the use of mathematical control theory to manage an economy. (Lucas and Sargent 1978, 50)

In planning for their 1978 presentation, they decided that it would be a “rhetorical piece... to convince others that the old-fashioned macro game is up...in a way which makes it clear that the difficulties are fatal”; its theme would be the “death of macroeconomics” and the desirability of replacing it with an “Aggregative Economics” whose foundation was “equilibrium theory” (Lucas in February 9, 1978 letter to Sargent). Their 1978 presentation was replete, as their discussant Bob Solow pointed out, with the planned rhetorical barbs of “wildly incorrect,” “fundamentally flawed,” “wreckage,” “failure,” “fatal,” “of no value,” “dire implications,” “failure on a grand scale,” “spectacular recent failure,” “no hope.” The empirical backdrop to Lucas and Sargent’s death decree on Keynesian economics was evident in the subtitle of the conference: “Persistence of High Inflation and High Unemployment.”

Although they seized the opportunity to comment on policy failure and the high misery-index economy, Lucas and Sargent shifted the macroeconomic court of judgment from the national economy to microeconomics. They fought a technical battle over the types of

restrictions used by modelers to identify their structural models. Identification-rendering restrictions were essential to making both the Keynesian and rational expectations models “work” in policy applications. Lucas and Sargent, while not fully forsaking the empirical explanation or achievement of a desirable policy outcome, shifted emphasis to a model’s capacity to incorporate optimization and equilibrium and to aggregate consistently rational individuals and cleared markets. In the macroeconomic history written by the victors, the Keynesian revolution and the rational expectations revolutions were both technical revolutions, and one could delineate the sides of the battle line in the second revolution by the nature of the restricting assumptions that enabled the model identification that licensed policy prescription. The rational expectations revolution, however, was also a revolution in the prime referential framework for judging macroeconomic model fitness for going forth and multiplying; the consistency of the assumptions – the equation restrictions - with optimizing microeconomics and mathematical statistical theory, rather than end uses of explaining the economy and empirical statistics, constituted the new paramount selection criteria.³⁹

It is no coincidence that in this narrative of economic equilibrium crafted in the Cold War era, Adam Smith’s invisible hand morphs into an invisible welfare-maximizing “planner” enforcing a communism of models and decreeing to individual agents the mutually consistent rules of action that become the equilibrating driving force. It is, however, ironic, that a decade-long government planning contract employing Carnegie professors and graduate students underwrote the two key modeling strategies for the Nobel-prize winning demonstration that the

³⁹ In a Mainly Marco blog post, Simon Wren-Lewis called Lucas and Sargent’s essay “After Keynesian Economics” “the New Classical manifesto” because of its methodological critique, <http://mainlymacro.blogspot.com/2014/07/rereading-lucas-and-sargent-1979.html>.

rationality of consumers purportedly renders government intervention to increase employment unnecessary and harmful.

Some of the new classical macroeconomists have been explicit about the narrowness of their revolution. For example, Sargent noted in 2008, “While rational expectations is often thought of as a school of economic thought, it is better regarded as a ubiquitous modeling technique used widely throughout economics.” In crafting an historical perspective on rise of the rational expectations approach to macroeconomic dynamics there are insights to be gained from highlighting the history of a modeling-strategy turn in economics rather than exclusively focusing on schools of thought. The acknowledgment of the momentum of modeling strategies, however, does not mean that historians have to also buy into the narrow historical perspective that sees the vanquished schools of thought, such as Keynesian economics, as mere modeling techniques. Also, honing in on the properties of mathematical models of systems might well have facilitated humans landing on the moon by 1969, but the 2008 financial and economic crisis highlighted the opportunity cost of myopia in macroeconomics that can arise if economists confine their focus to technical issues that arise from staring at models.

Table 1: Sveriges Riksbank Prizes in Economic Sciences in Memory of Alfred Nobel awarded to professors who had an affiliation with the Carnegie Institute of Technology

| Economic Laureates | Award Date | Nobel Citations- | Carnegie Dates |
|---|-------------------|---|--|
| Herbert Simon | 1978 | "for his pioneering research into the decision-making process within economic organizations" | 1949-2001 |
| Franco Modigliani | 1985 | "for his pioneering analyses of saving and of financial markets" | 1952-1960 |
| Robert Lucas | 1995 | "for having developed and applied the hypothesis of rational expectations, and thereby having transformed macroeconomic analysis and deepened our understanding of economic policy" | 1963-74 |
| Merton H. Miller | 1990 | for "pioneering work in the theory of financial economics" | 1953-1961 |
| Finn Kydland and Edward Prescott | 2004 | "for their contributions to dynamic macroeconomics: the time consistency of economic policy and the driving forces behind business cycles" | 1969-1973 PhD 1977-1995 prof 3 yrs 1960s PhD 1970-1978 prof |
| Thomas J. Sargent | 2011 | For "empirical research on cause and effect in the macroeconomy" | 1967-1969 |

Table 2 Selected Milestones for the Graduate School of Industrial Administration at Carnegie Institute of Technology

| | |
|---------|---|
| 1948 | The W.L. & Mary T. Mellon Foundation donates \$6 million to establish GSIA |
| 1949-50 | Herbert A. Simon & William W. Cooper initiate a Masters in Industrial Administration program stressing analytical decision-making Air Force's Project SCOOP grant for "Intra-Firm Planning & Control" begins |
| 1951-52 | For USAF, Cooper & Abraham Charnes apply linear programming for blending aviation fuel at Philadelphia Gulf Oil Refinery, Simon and Charles C. Holt apply servomechanism analysis to production control Ph.D. program in Mathematical & Industrial Economics begins. |
| 1952-53 | The Office of Naval Research's long-term grant for "Planning & Control of Industrial Operations" begins with focus on computer-implemented protocols for production, inventories, and work force at Pittsburgh Plate Glass Company |
| 1953-54 | The Ph.D. program in Administration starts |
| 1954-55 | Holt, Modigliani, Simon use quadratic cost function to obtain optimal, linear decision rules for firms. Simon's "A Behavioral Model of Rational Choice" argues that limitations on information access and computational resources lead entrepreneurs and consumers to approximate solutions for satisficing (as opposed to exact solutions for maximizing) |
| 1955-56 | IBM 650, first computer at CIT, is installed at GSIA |
| 1956-57 | In <i>Models of Man</i> , Simon names "Principle of Bounded Rationality" Simon and Allen Newell begin computer simulations to study human cognitive processes |
| 1959-60 | Publication of <i>Planning Production, Inventory, & Employment</i> by Holt, Modigliani, Muth & Simon. Crowning achievement of ONR project and canonical OR text using quadratic criterion functions, EWMA forecasting, linear decision rules of action. <i>JASA</i> publishes John Muth's "Optimal Properties of Exponentially Weighted Forecasts": EWMA = random walk + white noise |
| 1960-61 | <i>Econometrica</i> publishes Muth's "Rational Expectations & the Theory of Price Movements": Information is not wasted so firm's expectations = predictions of economic theory, thus expected price = equilibrium price" |
| 1961-62 | A new doctoral option in Operations Research is started. Newell and others establish an interdisciplinary program in Information Systems & Communications Science (GSIA, psychology, electrical engineering, & mathematics) precursor of the School of Computer Science |
| 1966 | Robert E. Lucas pens working paper "Optimal Investment with Rational Expectations" |
| 1971 | Lucas & Edward C Prescott publish "Investment under uncertainty" combining Muth's modeling strategy of rational expectations, Richard Bellman's dynamic programming protocol, and Harold Hotelling's (1931) assumption that the competitive industry in essence maximized consumer surplus and demonstrating the stabilizing qualities of a free market |

**Figure 1: Key Developments in GSIA Project on
Planning and Control of Industrial Operations for ONR**

1. *Quadratic cost function* for minimization criterion so mean value was sufficient w/o knowing probability distribution. (Holt, Modigliani, Muth, and Simon 1960, 81)

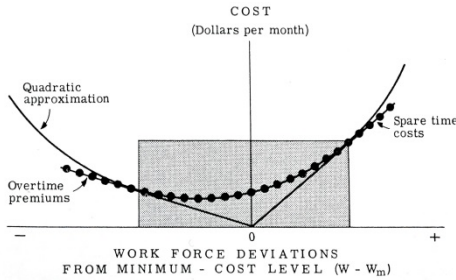


Fig. 3-10. Cost of deviations in work force from optimal levels. The shaded area is the range in which the quadratic approximates the cost function.

2. *Forecasting with exponentially weighted moving average* of past orders

$$\hat{o}_t = \hat{o}_{t-1} + w_e(o_{t-1} - \hat{o}_{t-1})$$

3. *Linear decision rule* (see box to right) Took only 3-man hours to compute with desk calculator then 5-minute calculation needed for monthly updates.

EXHIBIT I. PRODUCTION AND EMPLOYMENT DECISION RULES FOR PAINT FACTORY

$$P_t = \begin{Bmatrix} +.463 O_t \\ +.234 O_{t+1} \\ +.111 O_{t+2} \\ +.046 O_{t+3} \\ +.013 O_{t+4} \\ -.002 O_{t+5} \\ -.008 O_{t+6} \\ -.010 O_{t+7} \\ -.009 O_{t+8} \\ -.008 O_{t+9} \\ -.007 O_{t+10} \\ -.005 O_{t+11} \end{Bmatrix} + .993 W_{t-1} + 153. - .464 I_{t-1}$$

$$W_t = .743 W_{t-1} + 2.09 - .010 I_{t-1} + \begin{Bmatrix} +.0101 O_t \\ +.0088 O_{t+1} \\ +.0071 O_{t+2} \\ +.0054 O_{t+3} \\ +.0042 O_{t+4} \\ +.0031 O_{t+5} \\ +.0023 O_{t+6} \\ +.0016 O_{t+7} \\ +.0012 O_{t+8} \\ +.0009 O_{t+9} \\ +.0006 O_{t+10} \\ +.0005 O_{t+11} \end{Bmatrix}$$

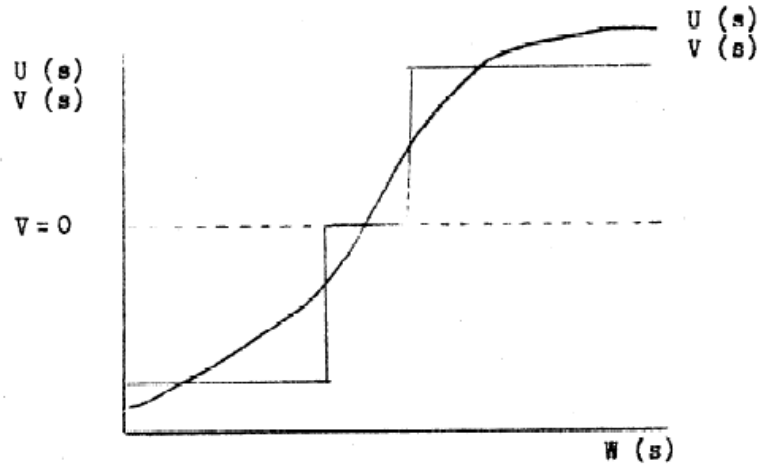
Where:

- P_t is the number of units of product that should be produced during the forthcoming month, t .
- W_{t-1} is the number of employees in the work force at the beginning of the month (end of the previous month).
- I_{t-1} is the number of units of inventory minus the number of units on back order at the beginning of the month.
- W_t is the number of employees that will be required for the current month, t . The number of employees that should be hired is therefore $W_t - W_{t-1}$.
- O_t is the forecast of number of units of product that will be ordered for shipment during the current month, t .
- O_{t+1} is the same for the next month, $t+1$; and so forth.

Anshen, Holt, Modigliani, Muth, and Simon 1958, 55

With permission from the *Harvard Business Review*

Figure 2. Simon's image of approximating mechanism to make do with limited computational capacity. (Source: Simon 1953, 6)



When a simplified $V(s)$, assuming only the values $\begin{Bmatrix} +1 \\ 0 \\ -1 \end{Bmatrix}$, is admissible, under the circumstances just discussed or under other circumstances, then a rational decision-process could be defined as follows:

- D. \mathcal{L} . Look for a subset $S' \subset S$ such that $V(s) = 1$ for all $s \in S'$.
- \mathcal{B} . Look for an $\underline{a} \in \underline{A}$ that maps on an $S_a \subset S'$.

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