

Stock-Market Expectations: Econometric Evidence that *both* REH and Behavioral Insights Matter

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ABSTRACT

The Rational Expectation Hypothesis models stock-market investors' expectations solely in terms of fundamentals, whereas behavioral finance supposes that expectations are driven by non-fundamental factors. Econometric evidence based on survey data indicates that both are relevant, but in ways that change over time. This evidence underscores the central importance of opening models to structural change and imposing discipline on econometric analysis through specification testing. Our findings support the novel hypothesis that *rational* market participants, faced with unforeseeable change, base their forecasts on both fundamentals – the focus of the REH approach - *and* the psychological and technical considerations underlying behavioral finance.

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1 Introduction

The Rational Expectations Hypothesis (REH) relates market participants' expectations to fundamental factors (such as company earnings and macro-economic variables). In a path-breaking paper, Shiller (1981) presented evidence that the REH-based present-value model is grossly inconsistent with persistent swings in stock prices. Behavioral-finance theorists have interpreted REH models' empirical difficulties as implying that participants' expectations, which drive stock-price swings, are driven by factors that are largely unrelated to fundamental considerations.¹

This paper makes two contributions. First, it presents econometric evidence that market participants base their forecasts on *both* fundamentals – the focus of the REH approach – and extrapolation, which is one of the non-fundamental factors underlying behavioral finance. Whereas fundamentals are a major driver of investors' expectations, the effect of extrapolation is short-lived, largely reversing itself after one month. Second, we show that the effects of both fundamentals and extrapolation vary in magnitude over time and that different fundamental factors matter in different time periods.

Although the first finding accords both REH and behavioral-finance insights a role in understanding investors' expectations, it is inconsistent with the key implications of each of the approaches taken separately. In particular, while the first finding supports REH models' focus on fundamentals, it contradicts these models' implication that bandwagon effects play no role in how market participants forecast outcomes. At the same time, it upends the *raison d'être* of behavioral-finance models, which assume that non-fundamental factors drive swings in stock and other asset prices.

The key to reconciling the seemingly contradictory REH and behavioral approaches is found in our econometric evidence that the structure of the relationship representing participants' expectations not only undergoes structural change, but does so at times and in ways that cannot be represented with a probabilistic rule. We estimate structural breaks to occur proximately to major events, including the Volcker disinflation and the bottom of the bear market in 1980, and the near-peak of the IT bubble in 1999. Such historical events are to some extent unique, with consequences that are, *ipso facto*, unforeseeable.² They thus engender so-called Knightian uncertainty – uncer-

¹For extensive surveys of the behavioral-finance approach see Shleifer (2000), Barberis and Thaler (2003), and references therein.

²For descriptive evidence that unforeseeable change moves stock prices, see Frydman

tainty that cannot “be reduced to an objective, quantitatively determined probability” (Knight, 1921, p. 321).

Frydman and Goldberg (2011, 2013a) hypothesized that, faced with unforeseeable change – change that cannot be represented with a probabilistic rule – *rational* participants would base their forecasts on fundamentals as well as draw on psychological and technical considerations. The econometric findings presented here are consistent with this hypothesis.

The paper joins a growing literature that relies on survey data of investors’ expectations to understand prices and risk premiums in asset markets.³ Prior to the use of these data, researchers relied on the indirect implications of alternative theoretical representations of expectations for asset-price movements. For REH, this typically involved imposing consistency within a specific model and testing its predictions for the quantitative co-movements between prices and fundamental factors, rather than investigating directly whether these factors drive investors’ expectations. As is well known, the “joint hypothesis” problem makes it very difficult to ascertain whether the failure of such studies to detect the role of fundamentals in asset-price movements arises from the invalid specification of the market’s expectation or from the wrong model of equilibrium returns.⁴

The availability of survey data has made it possible to test directly whether fundamental and/or behavioral considerations drive investors’ expectations. In order to carry out such a test, an investigator must decide which fundamental and non-fundamental variables to include in their empirical model for investors’ expectations. However, when it comes to testing the empirical relevance of the competing REH and behavioral-finance approaches, neither provides guidance concerning specific variables that an investigator might select.

The choice of the empirical model to test the predictions of the REH and behavioral-finance approaches is further complicated by the possibility that investors revise, at least intermittently, how they form expectations.⁵ This

et al. (2015).

³For recent studies using stock-market survey data and references to earlier literature, see Williams (2013) and Greenwood and Shleifer (2014).

⁴This observation has enabled proponents of REH to maintain that a better risk-premium model could overturn its empirical failures in asset markets. Although the search for such a risk-premium model has not been successful, many (notably Cochrane, 2011) continue the quest.

⁵The importance of structural change in an REH context has been emphasized by

might involve changes in the composition of the variables and/or changes in the weights attached to them when forecasting future stock prices.⁶ Such revisions imply that more than one empirical relationship would be required to represent investors' expectations during any sufficiently long sample period. Moreover, because the structural breaks are, at least in part, unforeseeable, probabilistic representations of change – regardless of whether they are based on REH, behavioral, or any other theory – cannot, *in principle*, guide where and how an investigator should look for change in the structure of their empirical model.

The foregoing arguments imply that – depending on the choice of variables and the procedure used to estimate structural change – there are many potential empirical specifications for a model of investor expectations. Given the absence from the REH and behavioral approaches of guidance concerning a model's specification and when and how it might undergo structural change, we adopt an econometric criterion.

In selecting an empirical model, we adhere to a key methodological principle: For an estimated relationship to serve as the basis for assessing the empirical relevance of alternative theoretical approaches, it should be well specified, in the sense of passing a battery of standard tests of specification error.⁷

The first stage of our investigation uses the Autometrics tree-search algorithm.⁸ This procedure implements the general-to-specific methodology, whereby all candidate variables are included from the outset.⁹ By design, Autometrics aims to achieve a well-specified model: it only adds to or re-

Lucas (1976) and Hamilton (1988, 1994). Frydman and Goldberg (2007, 2011) argue on theoretical and empirical grounds that the process underpinning the foreign exchange and equity markets undergoes structural change that is at least in part unforeseeable. For an overview of various approaches to modeling changes in expectations, see Frydman and Phelps (2013).

⁶Frydman et al. (2015) provide descriptive evidence that the number and composition of fundamental variables driving the stock-market participants expectations vary over time.

⁷For arguments concerning the importance in econometric analysis of achieving well-specified models, see Campos, Ericsson and Hendry (2005), Johansen (1995), and Juselius (2006).

⁸Automated model selection has advanced dramatically over the last decade and a half, owing to much-improved algorithms. The properties of the Autometrics procedure used here have been analyzed by Hendry and Krolzig (2001, 2005), and Doornik (2009).

⁹For a comprehensive overview of the arguments in favor of this methodology and automatic model selection, see Hendry and Doornik (2014).

moves a variable from a model if doing so improves the specification.

Autometrics also provides a way to diagnose the importance of structural change. To this end, Autometrics uses step indicator dummies to test for potential shifts in the constant term (Castle *et al.* 2015). We find that inclusion of these dummies improves model specification.

However, constraining the set of regressors (other than a constant term) and their parameters to remain time-invariant stops Autometrics before it can achieve a well-specified model. The residuals of the estimated specification are autocorrelated and heteroskedastic. This is what we would expect when representing expectations requires allowing for change in the set of model regressors and/or their parameters during the sample period.¹⁰ Consequently, we do not end our investigation with Autometrics. We consider the model specification selected by the algorithm as suggestive of the variables that might be relevant in specifying an empirical model of expectations.¹¹

We aim to achieve a well-specified model by allowing for change in all of its variables and parameters. This second step of our approach to model selection involves testing for structural change in the specification that relates investor expectations to the set of variables suggested by Autometrics. We then estimate separately a model for expectations involving these variables within each subperiod of statistical constancy, as judged by the structural-change test. The resulting piece-wise linear model is considered well specified if each of its linear segments passes a battery of standard tests of specification error.

Both Autometrics and the structural-change tests show that in order to achieve a well-specified model, we must allow its structure to change over time. This finding is inconsistent with the vast majority of existing models, which, regardless of whether they are based on REH or behavioral considerations, attempt to approximate investors' expectations with time-invariant structures.

Although any single structure eventually fails specification error tests, there may be protracted periods of time during which investors' expectations can be approximated with linear segments. Our empirical model for expectations involves three linear relationships, each of which is well specified.

¹⁰See Tabor (2013) for an econometric analysis of autocorrelation and the ARCH effects arising from ignoring structural change.

¹¹See Ericsson (2012) for the use of Autometrics as a diagnostic tool.

The empirical relevance of structural change implies that the model’s quantitative predictions vary across the linear segments. However, the model generates qualitative predictions that enable us to assess the empirical relevance of alternative theoretical approaches to modeling investors’ expectations.¹²

The estimates of the model that passes our rather stringent selection process indicate that the trend of at least one fundamental variable – the rate of interest and/or unemployment – is a major driver of investors’ expectations in every subperiod of approximate parameter constancy. In contrast, extrapolation plays a transient role in every linear segment.

We also find that the composition of the variables accounting for investors’ expectations differs across subperiods. Whereas both the interest rate and the unemployment rate drive expectations in one of the subperiods, only one of them matters in the other two.

Moreover, the estimated qualitative effects (signs of parameters) of fundamental variables appear to provide a sensible explanation of expectations during the subperiods approximated by each of the linear segments. According to the model, from 1963 to 1980, investors’ expectations appear to have been driven by both the interest rate and business-cycle effects. The bull market from the 1980s to 1999, however, appears to have been driven primarily by falling interest rates, while post-1999 expectations again focused on the macroeconomic outlook, as proxied by changes in unemployment.

The foregoing conclusions – that both extrapolation and fundamentals matter for understanding investors’ expectations – are based on a well-specified, piece-wise linear model. In order to underscore the central importance of opening finance models to structural change and imposing discipline on econometric analysis through specification testing, we compare our findings – that both extrapolation and fundamentals matter – with those of Shleifer and Greenwood (2014).

Relying on the same survey measure and the same set of fundamental variables as we do here, Shleifer and Greenwood (GS) found *no* role for fun-

¹²Generating quantitative predictions that span more than one segment requires further restrictions on change, for example, that change between segments is governed by a Markov switching rule. For a demonstration, see Hamilton (1994) and Frydman and Goldberg (2007). However, Stillwagon and Sullivan (2016) and Frydman *et al.* (2016a,b) show that, although a Markov switching or another probabilistic rule might provide an *ex post* approximation of the process during a sample period, this empirical characterization eventually fails to represent structural change in future periods.

damentals and thus concluded that investors' expectations are purely extrapolative. In Appendix B, we show that GS's econometric model is strongly rejected by each of the standard specification error tests. One of the important sources of this misspecification is the model's premise that the same set of variables, with unchanging parameter values, can represent how stock-market investors form expectations at each point in time. These findings illustrate how not allowing for structural change and not achieving a well-specified model may lead to unreliable conclusions concerning an essential question in finance and macroeconomics: Do REH and/or behavioral insights matter for understanding market participants' expectations?

The paper is organized as follows: Section 2 presents the survey data. Section 3 sketches our approach to model selection and formulates an unrestricted model. Section 4 shows that allowing for shifts in the constant term is not sufficient for Autometrics to produce a well-specified model. Section 5 carries out the key structural-change step of our approach to model selection and shows that allowing for such change yields a well-specified model. Section 6 relates our findings – that both fundamentals and extrapolation matter for investor expectations – to insights of the REH and behavioral-finance approaches.

Finally, section 7 discusses some broader implications of our findings. It offers a new interpretation of the significance of Shiller's (1981) seminal rejection of the REH-based present-value model as a way to represent how market participants form expectations on the basis of fundamentals. In contrast to the behavioral-finance approach, we suggest that Shiller's and many subsequent findings simply reflect the failure of REH to represent *rational* forecasting. Rather than implying irrationality, these findings are compatible with participants' rationality in real-world markets, where change is at least in part unforeseeable. The paper concludes with a call to economists to recognize the importance of unforeseeable change and the Knightian uncertainty that such change engenders. Beyond helping us to understand how to represent rational forecasting, opening models to such change promises to be the key to resolving many puzzles that have resisted explanation on rational grounds for decades.

2 Survey Data

The availability of survey data has made it possible to investigate directly whether the REH and/or behavioral-finance insights matter for understanding investor expectations. However, given sensitivity to question framing and interpretation, the evidence from survey data has been considered unreliable. It is argued that the surveys are either too noisy or unrepresentative to be useful, or that respondents are misinterpreting the question (Cochrane, 2011).

In a recent paper, Greenwood and Shleifer (2014) convincingly argue against this dismissal of stock-market survey data. Their paper shows that various measures of expected returns (seven sources of survey data in total) co-move strongly and positively, even though the surveys that underpin them rely on very different methodologies. Furthermore, they are highly correlated with mutual fund flows, indicating that they are representative of expectations that are relevant for market participants' decisions. This evidence buttresses Greenwood and Shleifer's argument that survey measures are not just "meaningless noise" (p. 715).

Our analysis of investor expectations relies on the Investors Intelligence Newsletter (II) survey, the longest available sample among those used by Greenwood and Shleifer (GS), spanning the period from 1963 to 2015. The II survey records the percentage of its participants' bullish, neutral, and bearish forecasts on a weekly basis. Given that most of the other variables are measured at monthly intervals, we use a monthly average. Moreover, following GS, we proxy investors' time- t expectation of "raw" stock returns (stock-price change) over the succeeding 12-month period, $t + 12$, with the difference between the proportion of investors who are bullish and bearish at t concerning stock prices at $t + 12$:

$$Exp_{t+12|t} = [\% \textit{bullish}_t - \% \textit{bearish}_t] \quad (1)$$

Measures computed according to (1) are not numerical observations of price changes expected by survey participants. However, GS show that these proxies are highly correlated with the shorter available sample from Gallup surveys which provide numerical forecasts of stock returns from September 1998 through May 2003.

3 Our Approach to Model Selection

Our examination of the empirical relevance of REH and behavioral-finance insights relies on the linear specification relating investors' expectations to fundamentals and extrapolation. We require that in order to serve as the basis for assessing these approaches, the empirical model must pass a battery of standard tests of specification error. We aim to achieve a well-specified model by remedying two sources of model misspecification: non-stationarity of the regressors and the common practice of constraining the model's structure to be time-invariant – that is, presuming that the same set of variables, with unchanging parameter values, can represent how investors form expectations at every point in time. We show that opening models to structural change is the key to achieving a well-specified model.

3.1 Autometrics

We use Autometrics to suggest a set of variables that might be relevant in modeling investor expectations. By allowing for structural breaks in a constant term, Autometrics is also useful in diagnosing the importance of structural change.

3.2 General Unrestricted Model

Autometrics relies on the general-to-specific methodology, whereby all potential variables are included from the outset. We use the Augmented Dickey-Fuller procedure (Said and Fuller, 1984) to test the non-stationarity of each of the candidate variables. As reported in Table A1 in Appendix A, the test finds the dependent variable – the survey measure for expectations – and the past rate of return on the S&P 500 basket of stocks to be stationary.

However, the ADF test does not reject the unit root in the interest rate, unemployment rate, consumption, and industrial production. The test also indicates that dividends and earnings have a linear trend. Consequently, in order to avoid misspecification and inference problems stemming from non-stationarity, we include only first differences among these regressors. Furthermore, two lags are included for each regressor and the dependent variable to address potential issues of serial correlation. Our initial unrestricted specification can be written as follows:

$$Exp_{t+12|t} = c + \sum_{j=1}^2 Exp_{t+12-j|t-j} + \sum_{i=1}^7 \sum_{j=0}^2 \beta_{i,j} X_{i,t-j} + \varepsilon_t \quad (2)$$

where $(c, \beta_{i,j}$ for $i = 1 \dots 7$, and $j = 0, 1, 2$) is a vector of parameters, $Exp_{t+12|t}$ denotes the proxy for the expected rate of return, and $X_i = [R_{t,t-12}, \Delta u_t, \Delta i_t, \Delta \ln(E_t), \Delta \ln(C_t), \Delta \ln(D_t), \Delta \ln(Y_t)]$ is a vector of variables including, respectively, the return over the past year to the S&P 500, unemployment, the one-year Treasury bill rate, earnings, consumption, dividends, and industrial production.

4 Results from Autometrics

Out of 23 candidate regressors (seven variables including two lags for each, and two lags of the dependent variable), Autometrics retains the past rate of return on the S&P 500, which represents extrapolation, and two fundamental variables – the change in unemployment and the Treasury bill rate – in the estimated empirical model of expectations.

Table 1
Autometrics Results for the II Survey Measure.

$Exp_{t+11 t-1}$	0.810 [37.84]
$R_{t,t-12}$	0.946 [13.67]
$R_{t-1,t-13}$	-0.888 [-12.74]
Δi_t	-4.259 [-5.11]
Δu_{t-2}	-3.809 [-1.75]
S:1966(04)	4.745 [2.70]
S:1979(09)	10.698 [3.24]
S:1980(06)	-11.464 [-3.54]
S:2015(05)	2.178 [4.06]

Caption: The second column displays the coefficient estimates with the t-values in brackets below.

We note that, although the unemployment rate is not significant at 5%, it was still retained by Autometrics. Apparently, the search algorithm's

attempt to drop this variable weakened the model’s specification.

Beyond suggesting which variables, out of many candidates, might be relevant in modeling investor expectations, Autometrics is useful in diagnosing structural change. We use a step indicator saturation (SIS) procedure (Castle et al., 2015) to allow for breaks in the constant term.¹³ Table 1 shows that Autometrics retains four of the step dummies indicating structural breaks.

However, constraining the set of regressors (other than a constant term) and their parameters to remain time-invariant, which the SIS procedure does, forces Autometrics to impute the model’s structural change to shifts in the constant term. As Table 2 below shows, this stops Autometrics’ search algorithm before it can achieve a well-specified model.

The diagnostics in Table 2 include the Lagrange multiplier test of serial correlation, labeled as AR (Godfrey 1978); tests of autoregressive heteroskedasticity, or ARCH (Engle 1982); normality (Doornik and Hansen 2008); heteroskedasticity (White 1980); and the RESET test of model misspecification (Ramsey 1969).

Table 2
Specification Tests after Autometrics

	Model 2
AR	0.0088
ARCH	0.3065
Normality	0.6291
Hetero	0.0044
RESET	0.0684

Caption: The figures represent the p-values for the respective tests of the model in table 1.

Table 2 makes clear that allowing for shifts in only the constant term produces a misspecified empirical model: its residuals are autocorrelated and heteroskedastic, failing the respective tests at the 1% level.

As we show next, this difficulty stems from constraining structural change to occur solely in the constant term. Once we allow for shifts in all other parameters, we achieve a well-specified, piece-wise linear model.

¹³Autometrics also uses an impulse indicator saturation procedure to control for outliers (see Hendry, Johansen, and Santos, 2008).

5 Structural Change as Model Selection

The key step in our approach to model selection is to allow for shifts in the parameters of all variables. Since our objective here is to examine whether REH and/or behavioral-finance insights matter for investor expectations, we do not seek to determine which specific variables matter. As long as our set of regressors includes some fundamental and behavioral variables, achieving a well-specified model enables us to test the relevance of these two approaches – widely considered mutually exclusive – to modeling investor expectations.

The specification suggested by Autometrics provides a parsimonious way to examine the empirical relevance of the REH and behavioral-finance approaches. To be sure, allowing for structural change in all parameters in a specification suggested by Autometrics might not result in a well-specified model.¹⁴ In that case, a model involving other variables might have to be used.

The specification suggested by Autometrics is well suited to examining the relevance of the REH and behavioral-finance approaches. This specification includes the past rate of return on the S&P 500 and its lag (representing the behavioral insight that investor expectations are extrapolative) and the two potentially important fundamental variables, unemployment and the interest rate (representing REH’s focus on fundamental factors’ key role in driving investor expectations).

We use the Bai and Perron (1998) procedure to estimate structural breaks in an empirical model.¹⁵ Table 3 presents the estimates for the timing of structural breaks.

¹⁴In this sense, requiring that the model be well specified disciplines the econometric investigation. It guards against an investigator’s use of structural-change analysis to select a model that might favor his preferred hypothesis. For an example of an empirical model in which allowing for structural change does not result in a well-specified model, see an earlier version of this paper (Frydman and Stillwagon, 2016). See also Appendix A below for an illustration of the danger that, unless models are well specified, investigators may erroneously conclude that fundamentals do not matter in investor expectations.

¹⁵Ericsson (2012) proposes an alternative to allow for changing betas over time, referred to as multiplicative indicator saturation (MIS), which uses regressors interacted with step indicators and conducts model selection with Autometrics. Kitov and Tabor (2016) investigate the properties of MIS. The application of MIS to the modeling of investor expectations is left for future research.

Table 3
Intervals of Approximate Parameter Constancy

63:03-80:03	80:04-99:10	99:11-15:06
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It is noteworthy that the timing of the structural breaks detected by the Bai-Perron test seems to coincide with major events including the regime change of the Volcker disinflation and the bottom of the bear market in 1980, and the near peak of the IT bubble in 1999. Such historical events are at least in part unique. Thus, the Bai-Perron test indicates the empirical relevance of unforeseeable change.¹⁶

5.1 Achieving a Well-Specified Model by Allowing for Structural Change

Using the timing of breaks indicated by the Bai-Perron test, we estimate the linear segments of approximate parameter constancy. We consider a piece-wise linear model resulting from testing for structural change to be well specified if *each* of its linear segments passes a battery of standard specification error tests. Subjecting the model estimated for each linear segment to such tests is the key step in our approach to model selection.

Columns 3-5 of Table 4 present the results of a battery of specification tests for each linear segment. For comparison, we also present the results of these tests for a time-invariant version of the model.

Table 4
Specifications Tests

	Time-invariant Model	63:03-80:03	80:04-99:10	99:11-15:06
AR	0.0012	0.0971	0.0705	0.1935
ARCH	0.0177	0.9827	0.4577	0.0922
Normality	0.0000	0.0425	0.3598	0.6705
Hetero	0.1327	0.4978	0.1702	0.0996
RESET	0.0015	0.0191	0.3638	0.3914

Caption: The figures represent the p-values for the respective tests and samples.

¹⁶This econometric evidence corroborates findings in Frydman *et al.* (2015) that participants pay attention to novel historical events in forming their expectations of stock prices.

Column 2 clearly shows that constraining the model parameters to be time-invariant results in gross misspecification. However, allowing for structural change provides a substantial remedy: as can be seen in columns 3-5, the AR, ARCH, Normality, and RESET tests turn from highly significant to insignificant.

6 The Model's Qualitative Predictions

Having shown that the piece-wise linear version of our model passes specification tests for all of its linear segments, we now examine whether it generates predictions concerning qualitative co-movements between investor expectations and regressor variables that represent determinants of these expectations. We use these predictions to assess the empirical relevance of REH and behavioral-finance insights for understanding investor expectations. To this end, Table 5 displays the estimates and test statistics for each linear segment.

Table 5
Parameter Estimates of Linear Segments

	63:03-80:03	80:04-99:10	99:11-15:06
$Exp_{t+11 t-1}$	0.711 [15.02]	0.769 [19.52]	0.777 [11.86]
$Exp_{t+10 t-2}$			-0.161 [-2.43]
$R_{t,t-12}$	1.209 [6.29]	0.997 [10.10]	0.690 [6.87]
$R_{t-1,t-13}$	-1.000 [-5.06]	-0.850 [-8.30]	-0.577 [-5.95]
Δi_t	-10.924 [-4.69]	-3.057 [-3.25]	4.888 [1.59]
Δu_{t-2}	-12.412 [-2.25]	0.748 [0.24]	-7.150 [-1.95]
c	3.138 [2.97]	-0.231 [-0.29]	8.089 [6.82]
Adj. R^2	0.707	0.775	0.724

Caption: The t-values are presented below coefficient estimates in brackets.

In order to facilitate the discussion of the results, we group the qualitative regularities predicted by a piece-wise linear model in Table 5 into three categories. The first concerns the degree of persistence to investor expectations.

The second and third involve predictions about the role of extrapolation and fundamentals in driving these expectations.

6.1 Persistence of Investor Expectations

Investor expectations tend to be persistent. The lagged dependent variable is highly significant, with t-values above 10. Remarkably, not only are all of the estimates of the coefficient for the proxy of lagged expectations positive; they also lie in a rather narrow range, between 0.7 and 0.8.¹⁷

6.2 Extrapolation

Table 5 shows that, as the behavioral-finance approach suggests, both the past return and its lag are significant in each sub-period. However, their estimates are approximately the same in magnitude and have the opposite sign. This means that it is the change of the past return, $\Delta R_{t,t-12} = (R_{t,t-12} - R_{t-1,t-13})$, rather than its level, that matters for investors. This change is highly positively correlated with a one-month change in price, ΔP_t (at 0.0000% level, with a correlation coefficient of 0.66). As both $\Delta R_{t,t-12}$ and ΔP_t are stationary and not particularly persistent (with autocorrelation coefficients of less than 0.3), the extrapolative component of expectations dissipates fairly quickly.

Thus, although our findings show that investor expectations are in part extrapolative, they also indicate that extrapolation did not drive a sustained swing in expectations, and thus in stock prices, during any of the sub-periods of our sample.

6.3 Fundamentals

Table 5 shows that, as the REH approach emphasizes, trends in fundamentals were primary drivers of swings in investor expectations and thus stock-price fluctuations during all three sub-periods.

¹⁷Building on insights of Keynes (1936), Frydman and Goldberg (2013b) provide a theoretical account of this persistence that is compatible with market participants' rationality.

6.3.1 Interest rate

The importance of the interest-rate variable is evident in Table 5. It has a highly significant and negative effect on investor expectations in the first two sub-periods.

6.3.2 Unemployment Rate

The unemployment rate is significant in the first and third sub-periods, during which it has a negative effect on expectations.

7 Concluding Remarks

Behavioral-finance theorists have interpreted the rejection by Shiller (1981) and others of the REH present-value model as implying that stock-market expectations are driven by factors that are largely unrelated to fundamentals. Because they embrace the belief that REH provides the way to represent rational forecasting, many economists have interpreted behavioral insights as implying that market participants are “less than fully rational” (Barberis and Thaler, 2003).

The finding presented here – that trends in fundamentals are a major driver of investor expectations – is inconsistent with this interpretation. Econometric evidence corroborates the extensive descriptive evidence in Frydman et al. (2015) concerning the factors that market participants consider relevant for understanding stock-market movements. As reported by Bloomberg’s market wraps, participants mention at least one of the fundamental factors as a mover of stock prices on nearly all (99.4%) of the trading days over a 17-year period (from January 1993 to December 2009). Psychological and technical considerations (such as extrapolation) were mentioned considerably less frequently. Nonetheless, their significance is obvious: Participants considered them relevant on roughly half of the trading days in the sample.

This descriptive and econometric evidence points to a new explanation of the failure of the REH-based present-value model: REH does not represent how rational, profit-seeking participants in real-world markets form expectations on the basis of fundamentals. Frydman and Goldberg (2013a) have traced the reason to REH’s core premise: In forming their expectations, market participants disregard all changes in the process underpinning outcomes

that cannot be foreseen with a probabilistic rule.

Frydman and Goldberg (2013a) have shown that opening models to unforeseeable change and the Knightian (non-probabilistic) uncertainty that such change engenders is the key to incorporating both REH and behavioral insights into representations of rational forecasting. As Keynes understood early on,

We are merely reminding ourselves that... our rational selves [are] choosing between alternatives as best as we are able, calculating where we can [on the basis of fundamentals], but often falling back for our motive on whim or sentiment or chance [Keynes, 1936, pp. 163, emphasis added].

This view of how rational participants forecast outcomes in real-world markets when faced with change that cannot be foreseen precisely, even in probabilistic terms, poses considerable challenges for both model-building and econometric methodology. The apparent empirical relevance of Knightian uncertainty and the results presented here suggest that addressing these challenges is an important objective of future research.

References

- [1] Bai, J. and P. Perron (1998), “Estimating and Testing Linear Models with Multiple Structural Changes,” *Econometrica*, 66(1), 47-78.
- [2] Barberis, N. and R. Thaler (2003), “A Survey of Behavioral Finance,” in Constandines, G., M. Harris and R. Stultz (eds.), *Handbook of the Economics of Finance*, Amsterdam: North Holland, 1053-1128.
- [3] Bauer M.D. and J.D. Hamilton (2015), “Robust Bond Risk Premia,” Working Paper.
- [4] Campos, J., N.R. Ericsson, and D.F. Hendry (eds.) (2005), *Readings on the General-to-Specific Modeling*, Chaltenham: Edward Elgar.
- [5] Castle, J.L., J. Doornik, D.F. Hendry, and F. Pretis (2015), “Detecting Location Shifts during Model Selection by Step-Indicator Saturation,” *Econometrics*, 3(2), 240-264.
- [6] Cochrane, J. (2011), “Presidential Address: Discount Rates,” *The Journal of Finance*, 66, 1047-1108.
- [7] Doornik, J.A. (1999), “Object-Oriented Matrix Programming using Ox,” London: Timberlake Consultants Press.
- [8] Doornik, J.A. and H. Hansen (2008), “An Omnibus Test for Univariate and Multivariate Normality,” *Oxford Bulletin of Economics and Statistics* 70(s1), 927-939.
- [9] Engle, R.F. (1982), “Autoregressive Conditional Heteroskedasticity, with Estimates of the Variance of United Kingdom Inflation,” *Econometrica*, 50(4), 987-1007.
- [10] Ericsson, N.R. (2012), “Detecting Crises, Jumps, and Changes in Regime,” Working paper, Board of Governors of the Federal Reserve System, Washington, D.C.
- [11] Frydman, R. and M.D. Goldberg (2007), *Imperfect Knowledge Economics: Exchange Rates and Risk*, Princeton University Press.

- [12] Frydman, R. and M.D. Goldberg (2011), *Beyond Mechanical Markets: Asset Price Swings, Risk, and the Role of the State*, Princeton University Press.
- [13] Frydman, R. and M.D. Goldberg (2013a), “The Imperfect Knowledge Imperative in Modern Macroeconomics and Finance Theory,” in Frydman, R. and E. S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton, N.J.: Princeton University Press, 140-165.
- [14] Frydman, R. and M.D. Goldberg (2013b), “Opening Models of Asset Prices and Risk to Non-Routine Change,” in Frydman, R. and E. S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton, N.J.: Princeton University Press, 207-247.
- [15] Frydman, R. and E.S. Phelps (2013), “Which Way Forward for Macroeconomics and Policy Analysis?,” in Frydman, R. and E. S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton, N.J.: Princeton University Press, 1-46.
- [16] Frydman, R. and J.R. Stillwagon (2016), “Stock-Market Expectations: Econometric Evidence that *Both* REH and Behavioral Insights Matter, Working paper, Institute for New Economic Thinking, <https://ineteconomics.org/ideas-papers/research-papers/stock-market-expectations-econometric-evidence-that-both-reh-and-behavioral-insights-matter>
- [17] Frydman, R., M.D. Goldberg, and N. Mangee (2015), “Knightian Uncertainty and Stock Price Movements: Why the REH Present-Value Model Failed Empirically,” *Economics E-Journal*, 9(24), 1-50.
- [18] Frydman, R., H. Frydman, J. R. Stillwagon and M. N. Tabor (2016a), “Markov Switching and Structural Change,” in preparation.
- [19] Frydman, R. , S. Johansen, A. Rahbek and M.N. Tabor (2016b), “Econometrics of Partly Open Models,” mimeo.
- [20] Godfrey, L.G. (1978), “Testing for Higher Order Serial Correlation in Regression Equations when the Regressors include Lagged Dependent Variables,” *Econometrica*, 46, 1303-1313.

- [21] Greenwood, R. and A. Shleifer (2014), “Expectations of Returns and Expected Returns,” *Review of Financial Studies*, 27(3), 714-746.
- [22] Hamilton, J. D. (1988), “Rational-Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates,” *Journal of Economic Dynamics and Control*, 12, 385-423.
- [23] Hamilton, J. D. (1994), *Time-Series Analysis*, Princeton, N.J.: Princeton University Press.
- [24] Hendry, D. F. and J. A. Doornik (2014), *Empirical Model Discovery and Theory Evaluation*, Cambridge, M.A: The MIT Press.
- [25] Hendry, D.F, S. Johansen and C. Santos (2008), “Automatic Selection of Indicators in a Fully Saturated Regression,” *Computational Statistics*, 23(2), 317-35.
- [26] Hendry, D.F. and H.M. Krolzig (2001), “Computer Automation of General-to-specific Model Selection Procedures,” *Journal of Economic Dynamics and Control* 25(6), 831-866.
- [27] Hendry, D.F. and H.M. Krolzig (2005), “The Properties of Automatic Gets Modelling,” *The Economic Journal*, 115(502), C32-C61.
- [28] Johansen, S. (1995), *Likelihood-based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press.
- [29] Juselius, K. (2006), *The Cointegrated VAR Model: Methodology and Applications*. Oxford University Press.
- [30] Keynes, J.M. (1936), *The General Theory of Employment, Interest and Money*, Harcourt, Brace and the World.
- [31] Kitov, O. and M. Tabor (2015), “Detecting Structural Breaks in Linear Models: A Variable Selection Approach using Multiplicative Indicator Saturation,” Working Paper, University of Copenhagen.
- [32] Knight, F.H. (1921), *Risk, Uncertainty and Profit*, Boston: Houghton Mifflin.

- [33] Lucas, R. E., Jr. (1976), "Econometric Policy Evaluation: A Critique," in Brunner, K. and A. H. Meltzer (eds.), *The Phillips Curve and Labor Markets*, Carnegie-Rochester Conference on Public Policy, Amsterdam: North-Holland.
- [34] Ludvigson and Ng (2009), "Macro Factors in Bond Risk Premia," *Review of Financial Studies* 22(12), 5027-5067.
- [35] Newey, W.K., and K.D. West (1987), "A Simple Positive Semi-Definite Heteroskedasticity and Autocorrelation-consistent Covariance Matrix," *Econometrica* 55(3), 703-708.
- [36] Ramsey, J.B. (1969), "Tests for Specification Errors in Classical Linear Least Squares Regression Analysis," *Journal of the Royal Statistical Society B*, 31(2), 350-371.
- [37] Said, S.E. and D.A. Dickey (1984), "Testing for Unit Roots in Autoregressive- Moving Average Models of Unknown Order," *Biometrika* 71(3), 599-607.
- [38] Shiller, R.J (1981), "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?," *American Economic Review* 71(3), 421-436.
- [39] Shleifer, A. (2000). *Inefficient Markets: An Introduction to Behavioral Finance*. Oxford: Oxford University Press.
- [40] Stillwagon, J.R. and P. Sullivan (2016), "Markov Switching Models of the Exchange Rate: Are There Only Two Regimes?" working paper.
- [41] Tabor, M.N. (2014) "Essays on Imperfect Knowledge Economics, Structural Change, and Persistence in the Cointegrated VAR Model" Ph.D. Thesis, University of Copenhagen.
- [42] White, H. (1980), "A Heteroskedastic-consistent Covariance Matrix Estimator and Direct Test for heteroskedasticity," *Econometrica*, 48(4), 817-38.

Appendix A

Table A1
Unit Root Tests

	ADF w/ trend	ADF w/o trend
$Exp_{t+12 t}$	0.0000	0.0000
$R_{t,t-12}$	0.0000	0.0000
$\ln(P_t/D_t)$	0.4759	0.5550
u_t	0.1599	0.0440
i_t	0.2678	0.4299
$\ln(Y_t)$	0.2500	0.2462
$\ln(E_t)$	0.000	0.6752
$\ln(D_t)$	0.0026	0.9212
$\ln(C_t)$	1.0000	0.0000

Caption: p-values from the Dickey Fuller tests of the null of a unit root

Appendix B

The Perils of Time-Invariant, Poorly Specified Models

Introduction

A recent paper by Greenwood and Shleifer (GS, 2014) uses survey data on investor expectations in the US stock market to examine what role extrapolation and fundamentals play in driving these expectations. GS present econometric analysis supporting the hypothesis that investor expectations are almost purely extrapolative and largely unrelated to fundamentals. They conclude that this empirical evidence is inconsistent with the REH theory and favors the behavioral approach. Although we rely here on one of the survey measures – the one spanning the longest time period by 24 years – and the subset of fundamentals that Greenwood and Shleifer (GS) use, we reach a very different conclusion.

This Appendix focuses on a largely overlooked problem that is inherent in modeling asset markets: misspecification arising from assuming away structural change. We show that GS’s model suffers from gross misspecification and that this is at least partly related to their attempt to represent investor expectations with a time-invariant set of variables and parameter values.

Greenwood and Shleifer’s Approach to Model Selection

GS use seven measures summarizing surveys of stock-market expectations. They estimate a number of alternative regression models that relate each of the measures to a proxy for extrapolation and a set of fundamental variables. They pick their preferred specification by selecting the subset of the variables that are consistently estimated, regardless of the survey measure used to proxy expectations.

Fundamentals are often statistically insignificant in the regressions estimated by GS. Moreover, whenever they are significant, variables that seem to matter differ across estimated specifications or have the wrong sign. By contrast, the proxies for extrapolative expectations are statistically significant and have the correct sign in specifications using different survey measures. Based on these results, GS pick as their preferred model a specification that accords no role to fundamentals and represents stock-market expectations as purely speculative. They conclude that these expectations “are well explained by two variables. First, when recent past returns are high, investors expect higher returns going forward. Second,...investor expectations are positively correlated with the price dividend ratio” (p. 729).

How GS Conclude that Fundamentals Do Not Matter

Our ADF tests, reported in Appendix A, show that some of the fundamental variables used by GS’s study and ours are non-stationary. GS do not address the inference problems implied by including non-stationary variables in a standard regression model. These problems are well known and have recently been shown to lead to unreliable inference in the context of asset markets. Bauer and Hamilton (2015) demonstrate that the bias of standard errors in models that include highly persistent or non-stationary variables could be very large and lead to erroneous conclusions concerning factors driving bond premia. For example, they show that “..the tests employed by Ludvigson and Ng (2009), which are intended to have a normal size of five percent, can have a true size of up to 54%” (Hamilton and Bauer, p.3).

Table A2 displays results for four specifications estimated by GS. In GS Models III1 and III2, the dependent variable is the proxy for the expected rate of return, based on the same III survey that we use in this paper. Models AAI1 and AAI2 use as their dependent variable a measure based on the survey by the American Association of Individual Investors which provides a shorter sample beginning in 1987.

Table A2
Estimates of GS Specifications

	GS Model II1	GS Model II2	GS Model AA1	GS Model AA2
C	-37.891 [-3.35]	2.106 [0.10]	-63.9351 [-4.58]	-77.785 [-2.58]
$R_{t,t-12}$	0.510 [7.00]	0.545 [8.30]	0.290 [5.38]	0.267 [4.44]
$\ln(P_t/D_t)$	13.242 [4.39]	2.958 [0.66]	18.076 [5.05]	20.517 [3.96]
i_t		-2.109 [-4.69]		0.655 [0.84]
u_t		1.328 [1.44]		0.357 [0.24]
$\Delta \ln(E_t)$		1.516 [0.10]		0.796 [0.07]

Caption: The dependent variable is [%bulls-%bears]. Sample from 1963:01-2015:06 for II and 1987:07-2015:06 for AA. Newey-West t-values in brackets.

We use somewhat different data sources for the regressors, and our sample ends about 3.5 years later. Nevertheless, the estimates in Table A2 are quite similar to those reported by GS in their Table 3 (p. 730).¹⁸ Although GS interpret the price-dividend ratio as an extrapolative variable, this variable captures both the effect of extrapolation, reflected in part in the price change, and the influence of fundamentals, which exert an effect on both the change in the stock price and dividends.¹⁹

As GS observe, extrapolation is quite evident. The effect of the past return is highly significant in all models with or without fundamentals, but the $\ln(P/D)$ becomes insignificant when fundamentals are included. By contrast, fundamentals do not seem to matter, let alone consistently, across models II2 and AA2. Earnings growth and unemployment are both insignificant, and the latter has the wrong sign. The interest rate is significant and has the correct (negative) sign in model II2. However, once the AA measure is used

¹⁸We use the Shiller data for the S&P 500, earnings, and dividends. Industrial production, the one-year Treasury rate, the U-3 unemployment rate, and personal consumption expenditure are from the FRED database.

¹⁹To facilitate comparison of our results with those by GS, we have also applied our model-selection procedure to the model that initially included $\Delta \ln(P_t/D_t)$. We found that even after using the change in the price-dividend ratio (to avoid problems stemming from non-stationarity) and allowing for structural change, our model-selection procedure could not achieve a well-specified model. See Frydman and Stillwagon (2015) for further details.

as a proxy for the expected return in model AA2, the interest rate switches sign to positive and loses significance completely. Based on such results across all seven measures, GS conclude that investor expectations are purely extrapolative.

Specification Tests

Reliance on standard t-ratios and correct signs of the effects of the extrapolative variables across all specifications are not sufficient to support GS’s conclusion that investor expectations “are well explained” as being largely unrelated to fundamentals.²⁰ The results of the specification tests reported in Table A3 strongly support this claim.

Table A3:
Specification Tests

	GS Model III	GS Model II2	GS Model AA1	GS Model AA2
AR	0.0000	0.0000	0.0000	0.0000
ARCH	0.0000	0.0000	0.0000	0.0000
Normality	0.6604	0.7263	0.2094	0.3442
Hetero	0.0000	0.0000	0.0098	0.0215
RESET	0.0000	0.0000	0.8502	0.8548

Caption: The figures represent the p-values for the respective tests and models.

These results indicate that regressions in Table A2 are grossly misspecified. Their errors are strongly autocorrelated, are heteroskedastic, and suffer from ARCH effects, implying that GS’s regressions do not adequately represent the role that either extrapolation or fundamentals play in driving investor expectations.

The sharp difference between our conclusions and that reached by GS illustrates the perils of the common practice of closing models to structural change. As the vast majority of existing models involving expectations constrain their structure to be time-invariant, this Appendix underscores the importance of opening theoretical models to structural change and requiring that their empirical counterparts be well specified.

²⁰GS rely on the approach used by Newey and West (1987) to correct the standard t-ratios for autocorrelation and heteroskedasticity with Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors. However, Bauer and Hamilton (2015) show that reliance on HAC corrected t-ratios does not adequately address the bias of standard errors in models that include highly persistent or non-stationary variables.